A Formalization of Objective and Subjective Time Ontologies

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Abstract

This paper presents a novel formalization of temporal notions and their classification. First, objective and subjective time perceptions are discussed from a philosophical and logical viewpoint. Then, all objective and subjective temporal concept types are identified, based on McTaggart's A- and B-series and Priorean tense logic, to which temporal events (or propositions) could be mapped. Time ontology is then defined according to a formalism previously introduced by the authors, together with a graphical representation of the proposed temporal concept type hierarchy. Temporal axioms and properties are finally identified, linking our logic with propositional logic*.

Keywords: Knowledge Representation, Ontology, Propositional Calculus, Temporal Logic.

1 Introduction

A time ontology is an ontology based on temporal notions. According to current sciences and philosophies, especially of Eastern origin, all objects and phenomena in the universe, whether they are humans, animals, plants, rocks, or a beautiful sunset, are transient, that is, they only exist within a certain timeframe. Since ontology, in its original definition, is a study of reality or existence of "things", it ensues that time is intrinsically part of any ontology. In addition, since temporal notions are sometimes born from subjective perceptions, a time ontology could include elements that are only valid to an individual, a group of individuals, or within a particular context. This paper attempts to formalize time ontology based on objective as well as subjective perceptions, drawing inspirations from logicians such as J.E. McTaggart, A.N. Prior, C. Lejewski, and others. Their theories are still considered valid nowadays, although recent developments have contributed to better formalization of temporal reasoning. The concepts of subjective and objective times have been discussed by philosophers but we believe that this paper presents for the first time a way to formalize them in an ontology. Our aim is to define an upper time ontology that could be later used in specific applications, such as to describe the temporal content of web pages or to build automated natural language translation engines.

Temporal logic is considered founded by A.N. Prior (1914-1969) (Lejewski 1959). His work and the history of time ontology are detailed by Øhrstrøm and Schärfe (2004). One of the early attempts to formalize time was undertaken by J.F. Allen (1984) with the introduction of a general theory of action and time, in which are categorized time-related actions, such as concurrent actions and their interactions, causation, intention, belief and plan, etc. Causal reasoning is also later expanded in other work (Stein and Morgenstern 1994). More recently, OWL-Time (http://www.isi.edu/~pan/OWL-Time.html), formerly DAML-Time, is a project aiming to develop a representative ontology of time that expresses temporal concepts and properties common to any formalization of time, and specifically, the temporal content of web pages and the temporal properties of web services (Hobbs et al. 2004). In OWL-Time, instant and interval are the only two main temporal entities, all other temporal notions being relations over these entities. Time ontologies formalizing instant and interval are also proposed by other authors (Zhou and Fikes 2000). It is interesting to note that Allen (1984) only accepts the concept of time interval but not that of instant or time-point, being considered instead as a "small interval" instead. Our formalization presupposes the concept of instant but does not explicitly elaborate that of interval, which we consider subsumed in the concepts of instant, time direction (i.e., the order between instants), and time continuity or density (i.e., the existence of other instants between any two instants). Other authors, such as Bittner (2002), embody those notions in the definition of a "time-line", which is isomorphic to the set of real numbers, of which a subset is a time interval. OWL-Time permits measurement (or quantification) of time, in terms of temporal unit, calendar and clock, although the concepts of present, past and future are only briefly discussed. OWL-Time could however be considered as an upper ontology on which other more specific or more detailed temporal ontologies (including ours) could be built.

This paper is organized as follows: Section 2 recalls the temporal notions introduced by McTaggart. Section 3 re-formalizes Priorean tense logic with linkages to McTaggart's notions and our proposed temporal concepts of objective and subjective times. Section 4 details our

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time ontology formalization. Section 5 identifies all temporal concept types and represents them in a tree-like hierarchy to assist with understanding. Section 6 details some temporal axioms that are fundamental to our theory, and derives temporal properties that relate our logic to propositional logic. And finally, Section 7 concludes our paper.

2 McTaggart Temporal Concepts

An important aspect of time is the question of its reality, first raised by J.E. McTaggart (1866-1925) in a 1908 article, in which the logician defines three categories of temporal notions: A-series, consisting of notions of past, present and future, B-series, with notions of "earlier than" and "later than" (in fact one notion can be deduced from the other), and C-series, which is B-series without embedded A-series (as we shall see, a B-series always implicitly embeds an A-series). McTaggart maintains that changes are only possible with an A-series, since, in an A-series, any present event was future (at a time in the past) and will be past (at a time in the future), while, in a B-series, if an event M is earlier than another event N, then M will forever be earlier than N, and thus there cannot be any changes in a B-series. On the other hand, from a psychological perspective, without changes, a human mind cannot form any notion of time. Therefore, a B-series cannot be used to define time, only an A-series can. However, a B-series cannot exist without an A-series because one cannot deny that changes must have occurred in order to affirm that an event is earlier or later than another in a B-series (e.g., two events may have occurred at the same location, thus implying that changes must have happened there). Suppose that there is a B-series without an embedded A-series (called a C-series in this case), that series can give us an idea about the order between the events in the series but cannot enable us to form a notion about what *time direction* really means. It is like being presented with two statements made at two different instants, say, in 1995 and in 2000. We know from the order between those two numbers that one statement is made *before* the other, but we cannot know which year we are in now and whether time progresses from 1995 to 2000, or the other way around. In fact, direction is the main characteristic of time and consequently C-series cannot be considered as an appropriate representation of time. So, if an A-series is essential to define time, it follows then that, if an A-series cannot be defined, neither can be time. The difficulty with an A-series is that it is impossible to accurately (i.e., logically or mathematically) define the "present" (or the "now"). Mathematically, a point in time could be defined as the convergence of a series of time intervals, one strictly enclosing the next, with the first time interval enclosing the present moment by some significant margin such that all observers can agree to. For example, if the current time is about 8:00 AM, the first time interval could be from 7:00 AM to 9:00 AM, and the next time interval could be from 7:30 AM to 8:30 AM, and so on. Since it would take an infinity of steps to converge, any "point in time", in particular the present, can only be a fictitious concept in the mathematical realm. Furthermore, since the present is not static (i.e., time is always "moving"), at some stage, it is impossible to objectively know whether the time interval being considered in the previous series still contains the present. Therefore, according to McTaggart and mathematical reasoning, time, in particular the present, is not *real.* This is also in line with quantum physics, according to which the existence or reality of a matter and the measurement of time could be quite subjective, although the perception of the present could be experienced by all human beings, with everyone generally being able to consciously perceive what he/she thinks of the very *present moment*.

In summary, the notion of time is subjective, or, at best, can only be considered as relatively objective, i.e., it is only objectively agreed to within certain contexts or bounds. Since anything *subjective* cannot be considered as *real* in the traditional science of physics, McTaggart's A-series is not real. And so is B-series as B-series implicitly includes A-series. While C-series may be real (since it can be objectively agreed to by all, e.g., no-one can deny that World War I happened *before* World War II), but, as discussed, it cannot be considered as an adequate representation of time. Therefore, any true ontology always relies on subjective temporal notions, whether explicitly or implicitly. In the following, whenever we refer to objectivity in temporal notions, we always mean objectivity in a relative sense, as absolute objectivity cannot be logically proven with time.

Furthermore, in modern logic, an event could be defined as an activity that involves an outcome (Allen 1984). It usually (but not necessarily) has two main attributes: a location and a time (Hobbs et al. 2004). However, in its general definition, an event is "something that happens at a given place and time" (as per http://wordnet.princeton.edu/). This means that an event is a record of some changes that occur at some place during some time. Stated differently, event is a result of perception of changes, which also gives rise to the notion of time. Event as change perception therefore precedes the formation of the notion of time (of that event). Event is real (as it can be objectively agreed to) while time is abstract. Thus, event defines time, and in turn, time is used to record event. This is why in our ontological formalization presented in this paper, event and time are closely linked, while in other theories (such as OWL-Time), they may be quite separate.

3 Objective and Subjective Time Ontologies

3.1 Objective Temporal Notions

A.N. Prior defines four "first-grade" temporal notions to express the ideas of "earlier" and "later", and their qualifications of *temporariness* and *permanency* (Øhrstrøm and Schärfe 2004). We propose to formalize those notions as four concept types, each of them is a function between (**P**x**T**) and **P**, where **T** is the Time Space and **P** is the Proposition Space, as used in propositional calculus (Klement 2006). The four functions could be defined as follows (where T(p,t) means "proposition p is true at instant t"):

(1) Anteriority (A): $A(p,t) \equiv_{def} \exists t' \leq t \; T(p,t')$ (paraphrase: p is true at time t or before)

(2) Posteriority (Po): $Po(p,t) \equiv_{def} \exists t' \geq t\ T(p,t')$ (paraphrase: p is true at time t or after)
(3) Permanent Anteriority (PeA): $PeA(p,t) \equiv_{def} \forall t' \leq t\ T(p,t')$ (paraphrase: p is always true at time t or before)
(4) Permanent Posteriority (PePo): $PePo(p,t) \equiv_{def} \forall t' \geq t\ T(p,t')$ (paraphrase: p is always true at time t or after)

Our definitions above rely on the notions of instant (t), time order (or time direction, which is the order relation "≤" between two instants), truth of a proposition at an instant (T(p,t)), and first-order logic (i.e., the universal and existential quantifiers "∀" and "∃"). These four definitions also formalize McTaggart's B-series notion. In addition, to complete A.N. Prior's first-grade notions, three further temporal notions could be derived from the above to express the ideas of temporariness and permanency. These three additional notions are independent of specific instants and are functions between **P** and **P**:

(5) Temporariness (T) = Anteriority or Posteriority, i.e., $T(p) \equiv_{def} \exists t\ A(p,t) \cup Po(p,t) = \exists t\ T(p,t)$ (paraphrase: p is temporarily (or sometime) true)
(6) Permanency (Pe) = Permanent Anteriority and Permanent Posteriority, i.e., $Pe(p) \equiv_{def} \forall t\ PeA(p,t) \cap PePo(p,t) = \forall t\ T(p,t)$ (paraphrase: p is permanently (or always) true)
(7) Discrete Permanency = Anteriority and Posteriority, i.e., $DPe(p) \equiv_{def} \forall t_0\ A(t_0,p) \cap Po(t_0,p) = \forall t_0 \exists t \exists t': t \leq t_0 \leq t', T(p,t) \cap T(p,t')$ (paraphrase: p is permanently and discretely true, i.e., at any moment, p is true before and after that moment. This is a new notion first introduced in this paper.)

A.N. Prior's second-grade temporal notions are first-grade notions, plus the notion of the *present* (or the *now*). In fact, Prior's second-grade notions are simply a more explicit expression of first-grade notions, if we accept McTaggart's argument that a B-series always implicitly embeds an A-series. We can now derive from the notion of the present four additional temporal notions. These are independent of specific instants and are functions between **P** and **P**:

(8) Future (F): $F(p) \equiv_{def} \exists t \geq Now\ T(p,t)$ (paraphrase: p will sometime be true)
(9) Past (Pa): $Pa(p) \equiv_{def} \exists t \leq Now\ T(p,t)$ (paraphrase: p was sometime true)
(10) Permanent Future (PeF): $PeF(p) \equiv_{def} \forall t \geq Now\ T(p,t)$ (paraphrase: p will always be true)
(11) Permanent Past (PePa): $PePa(p) \equiv_{def} \forall t \leq Now\ T(p,t)$ (paraphrase: p was always true)

The above 11 notions cover all objective temporal notions in our theory, which also encompass McTaggart's A- and B-series notions and A.N. Prior's first- and second-grade temporal notions.

Based on the above formal definitions, we can easily prove the following properties:

- Temporariness = Future or Past, i.e., $T(p) = F(p) \cup Pa(p)$ (paraphrase: p is temporarily true = p was or will be true)
- Permanency = Permanent Future and Permanent Past, i.e., $Pe(p) = PeF(p) \cap PePa(p)$ (paraphrase: p is permanently true = p was always and will always be true)
- Discrete Permanency subsumes Future and Past (see formal definition of the subsumption relation in Sect. 4), i.e., $DPe(p) > F(p) \cap Pa(p) = \exists t \exists t': t \leq Now \leq t', T(p,t) \cap T(p,t')$ (paraphrase: if p is permanently and discretely true, then in particular, p was true sometime in the past and will be true again sometime in the future).

3.2 Subjective Temporal Notions

The above temporal notions are *objective* as they imply that the time direction between two events ordered by the relation "≤" could be objectively perceived by all observers. However, as discussed earlier, time is subjective and therefore subjective temporal notions could be formally introduced as follows.

Subjective first-grade temporal notions are defined as functions between the domain set of **P**, **P**x**T**, or **P**x**T**x**O** (where **O** is the observer space), and the value set of **P**: (In the following, T(p,t,O) means "proposition p is true at time t according to observer O".)

(1) Subjective Anteriority (SA): $SA(p,t,O) \equiv_{def} \exists t' \leq t\ T(p,t',O)$ (paraphrase: p is true at time t or sometime before, according to observer O)
(2) Indeterminate Subjective Anteriority (ISA): $ISA(p,t) \equiv_{def} \exists t' < t\ \exists O\ T(p,t',O) = \exists O\ SA(p,t,O)$ (paraphrase: p is true at time t or sometime before, according to some observer)
(3) Subjective Permanent Anteriority (SPeA): $SPeA(p,t,O) \equiv_{def} \forall t' \leq t\ T(p,t',O)$ (paraphrase: p is always true at time t and before, according to observer O)
(4) Indeterminate Subjective Permanent Anteriority (ISPeA): $ISPeA(p,t) \equiv_{def} \forall t' \leq t\ \exists O\ T(p,t',O)$ (paraphrase: p is always true at time t and before, according to some observers – Note: there may be different observers at different times, i.e., $ISPeA(p,t) \neq \exists O\ SPeA(p,t,O)$)
(5) Subjective Posteriority (SPo): $SPo(p,t,O) \equiv_{def} \exists t' \geq t\ T(p,t',O)$ (paraphrase: p is true at time t or sometime after, according to observer O)
(6) Indeterminate Subjective Posteriority (ISPo): $ISPo(p,t) \equiv_{def} \exists t' \geq t\ \exists O\ T(p,t',O) = \exists O\ SPo(p,t,O)$ (paraphrase: p is true at time t or sometime after, according to some observer)
(7) Subjective Permanent Posteriority (SPePo): $SPePo(p,t,O) \equiv_{def} \forall t' \geq t\ T(p,t',O)$ (paraphrase: p is always true at time t and after, according to observer O)
(8) Indeterminate Subjective Permanent Posteriority (ISPePo): $ISPePo(p,t) \equiv_{def} \forall t' \geq t\ \exists O\ T(p,t',O)$ (paraphrase: p is always true at time t and after, according to some observers - Note: there may be different observers at different time, i.e., $ISPePo(p,t) \neq \exists O\ SPePo(p,t,O)$)
(9) Subjective Permanency (SPe): $SPe(p,O) \equiv_{def} \forall t\ T(p,t,O)$ (paraphrase: p is always true according to observer O)

(10) Indeterminate Subjective Permanency (ISPe): $ISPe(p) \equiv_{def} \forall t\ \exists O\ T(p,t,O)$ (paraphrase: p is always true according to some observers – Note: there may be different observers at different times, i.e., $ISPe(p) \neq \exists O\ SPe(p,O)$)

(11) Subjective Temporariness (ST): $ST(p,O) \equiv_{def} \exists t\ T(p,t,O)$ (paraphrase: p is sometime true according to observer O)

(12) Indeterminate Subjective Temporariness (ST): $IST(p) \equiv_{def} \exists t\ \exists O\ T(p,t,O) = \exists O\ ST(p,O)$ (paraphrase: p is sometime true according to some observer)

In the above, the notion of *subjectivity* expresses the idea that something is true according to one known observer while the notion of *indeterminate subjectivity* conveys that something is true according to some observer or observers, who are only known in particular contexts or particular instants of that observation.

When the notion of the present is added, we can define additional *subjective second-grade temporal notions* similarly:

(13) Subjective Future (SF): $SF(p,O) \equiv_{def} \exists t \geq Now\ T(p,t,O)$ (paraphrase: p is or will sometime be true according to observer O)

(14) Indeterminate Subjective Future (ISF): $ISF(p) \equiv_{def} \exists t \geq Now\ \exists O\ T(p,t,O) = \exists O\ SF(p,O)$ (paraphrase: p is or will sometime be true according to some observer)

(15) Subjective Permanent Future (SPeF): $SPeF(p,O) \equiv_{def} \forall t \geq Now\ T(p,t,O)$ (paraphrase: p is and will always be true according to observer O)

(16) Indeterminate Subjective Permanent Future (ISPeF): $ISPeF(p) \equiv_{def} \forall t \geq Now\ \exists O\ T(p,t,O)$ (paraphrase: p is and will always be true according to some observers - Note: there may be different observers at different times, i.e., $ISPeF(p) \neq \exists O\ SPeF(p,O)$)

(17) Subjective Past (SPa): $SPa(p,O) \equiv_{def} \exists t \leq Now\ T(p,t,O)$ (paraphrase: p is or was sometime true according to observer O)

(18) Indeterminate Subjective Past (ISPa): $ISPa(p) \equiv_{def} \exists t \leq Now\ \exists O\ T(p,t,O) = \exists O\ SPa(p,O)$ (paraphrase: p is or was sometime true according to some observer)

(19) Subjective Permanent Past (SPePa): $SPePa(p,O) \equiv_{def} \forall t \leq Now\ T(p,t,O)$ (paraphrase: p is and was always true according to observer O)

(20) Indeterminate Subjective Permanent Past (ISPePa): $ISPePa(p) \equiv_{def} \forall t \leq Now\ \exists O\ T(p,t,O)$ (paraphrase: p is and was always true according to some observers - Note: there may be different observers at different times, i.e., $ISPePa(p) \neq \exists O\ SPePa(p,O)$)

(21) Subjective Discrete Permanency = Subjective Anteriority and Subjective Posteriority: $SDPe(p,O) \equiv_{def} (\forall t_0\ SA(p,t_0,O) \cap SPo(p,t_0,O)) = (\forall t_0\ \exists t\ \exists t'\text{: } t \leq t_0 \leq t',\ T(p,t,O) \cap T(p,t',O))$ (paraphrase: At any instant t, p is true before and after t according to observer O, in particular, p was true and will be true again according to observer O). Note that Subjective Discrete Permanency subsumes Subjective Past and Subjective Future, i.e., $SDPe(p,O) > (SPa(p,O) \cap SF(p,O))$ since the right part of the equation is equal to $(\exists t\ \exists t'\text{: } t \leq Now \leq t',\ T(p,t,O) \cap T(p,t',O))$

(22) Indeterminate Subjective Discrete Permanency = Indeterminate Subjective Anteriority and Indeterminate Subjective Posteriority, i.e., $ISDPe(p) \equiv_{def} (\forall t_0\ \exists O\ \exists O'\ SA(p,t_0,O) \cap SPo(p,t_0,O')) = (\forall t_0\ \exists O\ \exists O'\ \exists t\ \exists t'\text{: } t \leq t_0 \leq t',\ T(p,t,O) \cap T(p,t',O'))$ (paraphrase: if p is discretely and permanently true according some observers, then in particular, p was true and will be true again according to some observers – Note: there may be different observers at different times, i.e., $ISDPe(p) \neq \exists O\ SDPe(p,O)$). Also note that Indeterminate Subjective Discrete Permanency subsumes Indeterminate Subjective Past and Indeterminate Subjective Future, i.e., $ISDPe(p) > (\exists O\ \exists O'\ SPa(p,O) \cap SF(p,O'))$ since the right part of the equation is equal to $(\exists t\ \exists t'\ \exists O\ \exists O'\text{: } t \leq Now \leq t',\ T(p,t,O) \cap T(p,t',O'))$

The above 22 definitions cover all subjective temporal notions in our formalism, which also extend McTaggart's A- and B-series notions and A.N. Prior's first- and second-grade temporal notions, into subjectivity.

4 Proposed Ontology Formalization

Nguyen and Corbett (2003, 2006) define an *ontology* as a semantically consistent subset of a *canon*, which is in essence a mapping of a real world onto an abstract world. In this paper, to simplify and without loss of generality, we consider these two notions identical.

In our formalism, a *time ontology* (or *time canon*) could be formally defined as a 5-tuple $\boldsymbol{K} = (T, I, <, conf, B)$ in which:

(1) T is the set of temporal concept and relation types, i.e., $T = T_C \cup T_R$ where:
   (a) T_C is the set of temporal concept types, consisting of 11 objective and 22 subjective temporal notions as listed above.
   (b) T_R is the set of temporal relation types, consisting of 3 elements similar to the three main logical connectives of propositional calculus, i.e., negation ($\neg$), conjunction ($\cap$), and disjunction ($\cup$) (Smith 2003), defined as follows:
   - $\neg$ is a unary relation over T_C , i.e. $\neg$: $T_C \rightarrow T_C$ with $\forall c \in T_C$ the value $\neg(c)$ (simply written as $\neg c$) is a temporal concept type defined over the same domain set as c, i.e., $\forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}$
     - if c is defined over **P** only, then $(\neg c)(p) = \neg(c(p))$
     - if c is defined over **PxT**, then $(\neg c)(p,t) = \neg(c(p,t))$
     - if c is defined over **PxTxO**, then $(\neg c)(p,t,O) = \neg(c(p,t,O))$
   - $\cap$ is a binary relation over T_C x T_C , i.e., $\cap$: T_C x $T_C \rightarrow T_C$ with $\forall c,c' \in T_C$ the value $\cap(c,c')$ (simply written as $c \cap c'$) is a temporal concept type defined over the largest of the 2 domain sets used by c and c', i.e.,

- if c and c' are both defined over **P** only, or over **P**x**T,** or over **P**x**T**x**O**, then so is $c \cap c'$ with: $\forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}$
  $(c \cap c')(p) = c(p) \cap c'(p)$
  or $(c \cap c')(p,t) = c(p,t) \cap c'(p,t)$
  or $(c \cap c')(p,t,O) = c(p,t,O) \cap c'(p,t,O)$
- if there is a difference in the domain sets of c and c', then $c \cap c'$ is defined over the largest domain set of the two, e.g., if c is defined over **P** only and c' is defined over **P**x**T**x**O**, then $c \cap c'$ is defined over **P**x**T**x**O** with: $\forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}$
  $(c \cap c')(p,t,O) = c(p) \cap c'(p,t,O)$

- $\cup$ is defined similarly to $\cap$.

(2) *I* is the set of *instances* of temporal concept types in T_C. I consists of all *atomic* propositions that contain temporal notions, i.e., temporal propositions that cannot be further divided into sub-propositions connected by any of the four logical connectives of propositional calculus: "and", "or", "not", and "implication". For example, the proposition "it was hot yesterday but it will be cooler tomorrow" could be considered as two atomic temporal propositions: "it was hot yesterday" and "it will be cooler tomorrow" connected by the logical connective "and" (i.e., "$\cap$"). Note that our definition of *temporal proposition* is what OWL-Time calls *eventuality* or *event*.

(3) "<" is the subsumption relation in *T*, defined as a binary relation between temporal concept types or between temporal relation types, such that the first type is *semantically entailed* by the second, e.g., the relation "$b < a$" or "$a > b$" between two temporal concept types *a* and *b* means: "*a* semantically entails *b*". This subsumption relation is based on the *semantic entailment* relation of propositional calculus (normally represented by the symbol "$\models$") (Smith 2003). As we shall see, in some cases, *semantic entailment* in our subsumption relation also means *syntactic proof* (normally represented by the symbol "$\vdash$") (Smith 2003). Formally, "<" can be defined as follows:

- Subsumption relation in T_C:

$\forall c,c' \in T_C$ we have: c > c' if and only if :

a) If the domain set of c' is larger than, or equal to, that of c, then the semantic entailment relation between the propositions transformed by c and c' (i.e., the values of the functions c and c') must be true for all instances of the common domain set (between c and c'), and for all instances of each extra dimension of the domain set of c', i.e.,

$c>c' \equiv_{def} \forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}\ c(p) \models c'(p) | c'(p,t) | c'(p,O) | c'(p,t,O)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}\ c(p,t) \models c'(p,t) | c'(p,t,O)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \forall O \in \mathbf{O}\ c(p,t,O) \models c'(p,t,O)$

b) If the domain set of c is larger than that of c', then the semantic entailment relation between the propositions transformed by c and c' (i.e., the values of the functions c and c') must be true for all instances of the common domain set (between c and c'), and for at least one instance of each extra dimension of the domain set of c, i.e.,

$c>c' \equiv_{def} \forall p \in \mathbf{P}\ \exists t \in \mathbf{T}\ c(p,t) \models c'(p)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \exists O \in \mathbf{O}\ c(p,O) \models c'(p)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \exists t \in \mathbf{T}\ \exists O \in \mathbf{O}\ c(p,t,O) \models c'(p)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \forall t \in \mathbf{T}\ \exists O \in \mathbf{O}\ c(p,t,O) \models c'(p,t)$

or $c>c' \equiv_{def} \forall p \in \mathbf{P}\ \forall O \in \mathbf{O}\ \exists t \in \mathbf{T}\ c(p,t,O) \models c'(p,O)$

In the above, the symbol "|" means "logical or", e.g., "c(p)|c(p,t)|c(p,t,O)" means "c(p), c(p,t), or c(p,t,O), depending on the domain set of c". Note that in the above, condition a) is generally used to determine that an objective concept subsumes a subjective concept of the same nature (such as PeA > SPeA), while condition b) is generally used to determine the subsumption relation between two concepts of the same category (i.e., both objective or both subjective, such as SA > ISA).

- Subsumption relation in T_R:

The subsumption relation "<" among the temporal relation types in T_R could be formally defined as: $\forall r,r' \in T_R\ \ r > r' \equiv_{def} \forall c,c' \in T_C\ \ r(c,c') > r'(c,c')$ with the relation "r(c,c') > r'(c,c')" defined similarly to the relation "<" between two elements of T_C as above. In fact, since there are only 3 temporal relation types: $\neg$, $\cap$, and $\cup$, it can be proven that the only subsumption relation in T_R is: "$\cap > \cup$". Indeed, $\forall c,c' \in T_C$, we have (assuming that c and c' are defined over **P** only, to simplify):

$(c \cap c') > (c \cup c')$

or $\forall p \in \mathbf{P}\ (c \cap c')(p) \models (c \cup c')(p)$

or $\forall p \in \mathbf{P}\ (c(p) \cap c'(p)) \models (c(p) \cup c'(p))$

The last statement is true because in propositional calculus, "any two propositions that are jointly true always imply that either proposition is true".

(4) *conf* is the "conformity" relation, defined between the set of all non-tautological temporal concept type instances (denoted as $I \backslash \{*\}$) and the set of all temporal concept types T_C, i.e., *conf*: $I \backslash \{*\} \rightarrow T_C$ where {*} represents the set of all *tautologies* in propositional calculus. The *conf* function expresses the idea that any atomic temporal proposition, except a tautology, can be associated with a temporal concept type. For example, the temporal proposition: "The phenomenon p has been observed throughout the ages" can be translated as "$\forall t \leq Now\ \exists O\ T(p,t,O)$", or p can be associated with the "Indeterminate Subjective Permanent Past" concept type of T_C (i.e., if we call that statement q, then $q \in I \backslash \{*\}$ and *conf*(q)=ISPePa). We should distinguish that statement with: "Someone has always observed the phenomenon p", translated as "$\exists O\ \forall t \leq Now$: $T(p,t,O)$", or p is an instance of the "Subjective Permanent Past" concept type. Similarly, the statement: "The truth p will be revealed to all in the future" could be translated as "$\exists t \geq Now\ \forall O\ T(p,t,O)$", or "$\exists t \geq Now\ T(p,t)$", or "p is a Future truth" (i.e., p is an instance of the "Future" temporal concept type), while the statement: "Someone will

know the truth p" could be translated as "∃O ∃t≥Now T(p,t,O)", or "p is a Subjective Future" truth (i.e., p is an instance of the "Subjective Future" temporal concept type). Note that in OWL-Time, the relations between propositions and times are *atTime*(e,t|T) and *holds*(e,t|T) (meaning "the proposition or event e holds at instant t or during interval T"). These relations are similar to our *conf* function. OWL-Time separates the event (or proposition) ontology from the time ontology. (In fact, *atTime* is a relation in the time ontology while *holds* is a relation in the event ontology, although both have the same semantics in OWL-Time.) In our formalism, we link them together because as discussed earlier we consider that propositions and events are part of the real world while time is part of an abstract world, and an ontology is a formal attempt to link those two worlds (Nguyen et al. 2006).

(5) *B* is the *Canonical Basis* function, defined between T_R and the set of all subsets of T_C (denoted as $\varphi(T_C)$), i.e., B: $T_R \rightarrow \varphi(T_C)$. *B* expresses the "usage pattern" (or "canonical basis") of each temporal relation type, that is, it defines which temporal concept types can be used in each temporal relation type. In our time ontology, based on the above definitions of T_C and T_R there is no restriction and any temporal concept type can be used with any temporal relation type. This is similar to propositional calculus, in which the relations $\neg$, $\cap$, and $\cup$ can be used with any propositions.

Finally, note that our formalism could be considered as a *meta-logic* since it is defined on top of propositional logic.

5 Representation of Time Ontologies

In the objective time ontology, the previously identified 11 objective temporal concepts could be *syntactically proven* to be linked by 10 subsumption relations, based on their predicate formulae specified in Section 3. (More correctly, those 10 relations are 10 *supertypes* (Sowa 1984), as some relations are between more than two concepts.) This means that our temporal subsumption relation ("<") that is based on *semantic entailment* can also be said to be based on *syntactic proof* (Smith 2003):

1. Anteriority > Temporariness
2. Discrete Permanency > Anteriority, Future, Past, Posteriority
3. Future > Temporariness
4. Past > Temporariness
5. Permanency > Discrete Permanency, Permanent Anteriority, Permanent Future, Permanent Past, Permanent Posteriority
6. Permanent Anteriority > Anteriority
7. Permanent Future > Future
8. Permanent Past > Past
9. Permanent Posteriority > Posteriority
10. Posteriority > Temporariness

Figure 1 (drawn with a tool built by the authors (Nguyen et al. 2006)) shows the objective temporal concept type hierarchy. Note that 'permanency' is at the top of the hierarchy while 'temporariness' is at its bottom. (Also note that in all figures, concept names between parentheses are *co-references* (Sowa 1984).)

Similarly, it can be syntactically proven that there are 21 (n-ary) subsumption relations among the 22 subjective temporal concept types, forming a hierarchy represented in Figure 2 (with acronyms used in order to reduce the figure size). Note that 'subjective permanency' is at the top of the hierarchy while 'indeterminate subjective temporariness' is at its bottom.

In the combined objective-subjective ontology, we can identify additional subsumption relations linking objective with subjective concepts, based on the formal definition of the subsumption relation in Sect. 4. In general, an objective concept semantically entails (or subsumes) the subjective concept of the same nature, since "an objectively true proposition" means "a proposition true to all observers". Also, no subjective concept type can subsume an objective concept type due to the extra observer dimension needed in the former. Therefore, the following 11 additional subsumption relations forms the complete list of objective-subjective relationships (with acronyms used for legibility):

1. A > SA
2. Po > SPo
3. F > SF
4. Pa > SPa
5. T > ST
6. DPe > SDPe
7. Pe > SPe
8. PeA > SPeA
9. PePo > SPePo
10. PeF > SPeF
11. PePa > SPePa

Finally, if we add the above 11 objective-subjective relations to the previous 10 objective and 21 subjective relations, we obtain a total of 42 relations, which can be consolidated into 32 (n-ary) subsumption relations (after relation consolidation (Nguyen et al. 2006)) between the 33 objective and subjective temporal concept types. They can be fully listed as follows:

1. A > SA, T
2. DPe > A, F, Pa, Po, SDPe
3. F > SF, T
4. ISA > IST
5. ISDPe > ISA, ISF, ISPa, ISPo
6. ISF > IST
7. ISPa > IST
8. ISPe > ISPeA, ISPeF, ISPePa, ISPePo
9. ISPeA > ISA
10. ISPeF > ISF
11. ISPePa > ISPa
12. ISPePo > ISPo
13. ISPo > IST
14. Pa > SPa, T
15. Pe > DPe, PeA, PeF, PePa, PePo, SPe
16. PeA > A, SPeA
17. PeF > F, SPeF
18. PePa > Pa, SPePa
19. PePo > Po, SPePo
20. Po > SPo, T

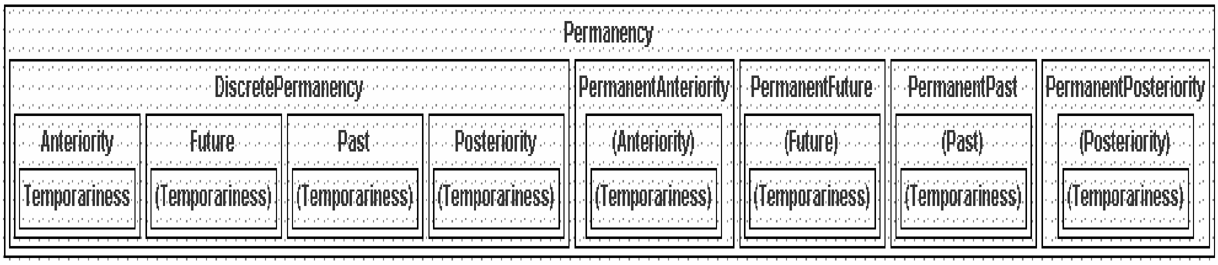

Fig. 1. Objective Temporal Concept Type Hierarchy

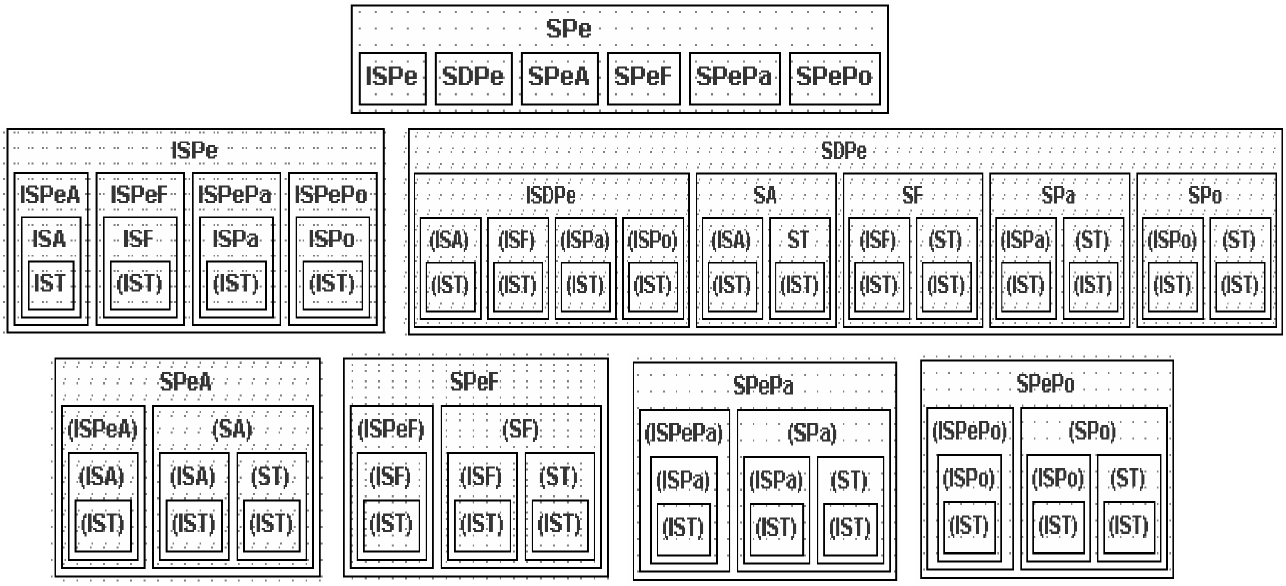

Fig. 2. Subjective Temporal Concept Type Hierarchy

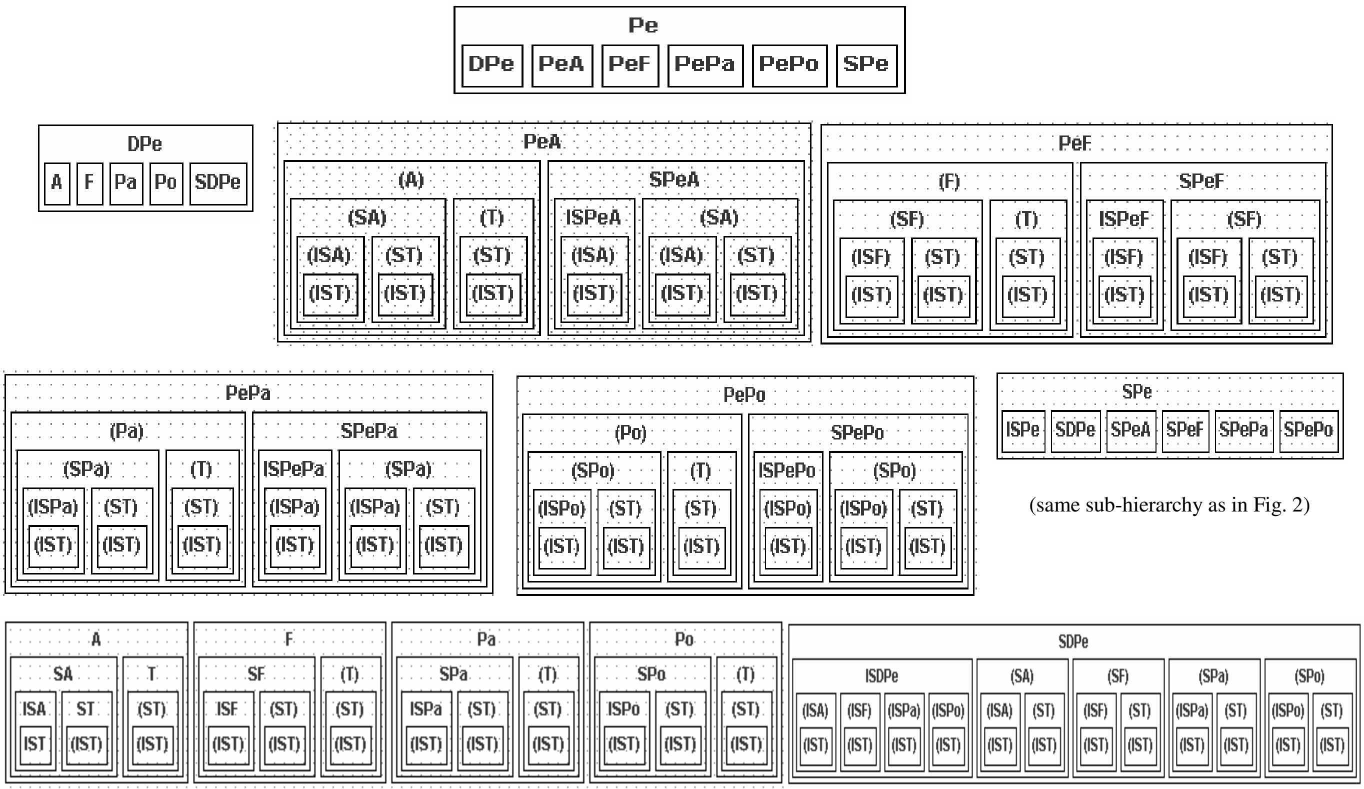

Fig. 3. Combined Temporal Concept Type Hierarchy

21. SA > ISA, ST
22. SDPe > ISDPe, SA, SF, SPa, SPo
23. SF > ISF, ST
24. SPa > ISPa, ST
25. SPe > ISPe, SDPe, SPeA, SPeF, SPePa, SPePo
26. SPeA > ISPeA, SA
27. SPeF > ISPeF, SF
28. SPePa > ISPePa, SPa
29. SPePo > ISPePo, SPo
30. SPo > ISPo, ST
31. ST > IST
32. T > ST

Based on these subsumption relations, the combined temporal concept type hierarchy could be represented in Figure 3. Note that ‘permanency’ (coming from the objective ontology) is at the top of the hierarchy while ‘indeterminate subjective temporariness’ (coming from the subjective ontology) is at its bottom, as one may intuitively expect in light of the earlier remarks on objective-subjective subsumption relations.

6 Temporal Axioms and Properties

In this section, we will attempt to identify key axioms and properties in our temporal logic. We call our axioms *Truth Axioms* because they express the semantics of the truth functions T(p,t) and T(p,t,O).

6.1 Temporal Axioms

- Truth Axiom 1: $\forall p \;\; p \Rightarrow (\forall c \;\; c(p))$

(paraphrase: If a proposition is true, then it is true under any temporal concept type.)

This axiom is the most basic and fundamental in our theory. It simply states that if a proposition is true without any temporal qualification, then it is supposed to be *permanently* true. And since it is permanently true and permanency is at the top of our temporal concept type hierarchy, it is true with any subtype of permanency, i.e., true with any other temporal concept type.

- Truth Axiom 2:

(2a) $T(p \Rightarrow q,t) = (T(p,t) \Rightarrow T(q,t))$

and $T(p \Rightarrow q,t,O) = (T(p,t,O) \Rightarrow (T(q,t,O))$

(paraphrase: If at time t (and according to observer O), “p implies q” is true, then “p is true at t (and according to observer O)” implies “q is true at time t (and according to observer O)”, and vice-versa.)

(2b) $T(p \cap q,t) = (T(p,t) \cap T(q,t))$

and $T(p \cap q,t,O) = (T(p,t,O) \cap (T(q,t,O))$

(paraphrase: If at time t (and according to observer O), both p and q are true, then both “p is true at time t (and according to observer O)” and “q is true at time t (and according to observer O)” are true, and vice-versa.)

(2c) $T(p \cup q,t) = (T(p,t) \cup T(q,t))$

and $T(p \cup q,t,O) = (T(p,t,O) \cup (T(q,t,O))$

(paraphrase: If at time t (and according to observer O), either p or q is true, then either “p is true at time t (and according to observer O)” or “q is true at time t (and according to observer O)” is true, and vice-versa.)

(2d) $\neg T(p,t) = T(\neg p,t)$

and $\neg T(p,t,O) = T(\neg p,t,O)$

(paraphrase: If at time t (and according to observer O), p is not true, then it is true that “p is not true at time t (and according to observer O)”, and vice-versa.)

- Truth Axiom 3:

(3a) $(\forall t \;\; T(p \Rightarrow q,t)) = ((\forall t \; T(p,t)) \Rightarrow (\forall t' \; T(q,t')))$

and $(\forall t \; \forall O \; T(p \Rightarrow q,t,O)) = ((\forall t \; \forall O \; T(p,t,O)) \Rightarrow (\forall t' \; \forall O' \; T(q,t',O')))$

(paraphrase: If at any time (and according to any observer), “p implies q” is true, then “p is true at all times (and according to all observers)” implies “q is true at all times (and according to all observers)”, and vice-versa.)

(3b) $(\forall t \;\; T(p \cap q,t)) = ((\forall t \; T(p,t)) \cap (\forall t' \; T(q,t')))$

and $(\forall t \; \forall O \; T(p \cap q,t,O)) = ((\forall t \; \forall O \; T(p,t,O)) \cap (\forall t' \; \forall O' \; T(q,t',O')))$

(paraphrase: If at any time (and according to any observer), both p and q are true, then both “p is true at all times (and according to all observers)” and “q is true at all times (and according to all observers)” are true, and vice-versa.)

(3c) $((\forall t \; T(p,t)) \cup (\forall t' \; T(q,t'))) \Rightarrow (\forall t \;\; T(p \cup q,t))$

and $((\forall t \; \forall O \; T(p,t,O)) \cup (\forall t' \; \forall O' \; T(q,t',O'))) \Rightarrow (\forall t \; \forall O \;\; T(p \cup q,t,O))$

(paraphrase: If either “p is true at all times (and according to all observers)” or “q is true at all times (and according to all observers)” is true, then at any time (and according to any observer), either p or q is true.) Note that the converse of this Truth Axiom does **not** hold.

(3d) $(\forall t \;\; \neg T(p,t)) = (\forall t \; T(\neg p,t))$

and $(\forall t \; \forall O \; \neg T(p,t,O)) = (\forall t \; \forall O \; T(\neg p,t,O))$

(paraphrase: If at all times (and according to all observers), p is not true, then it is true at all times (and according to all observers) that “p is not true (at those times (and according to those observers))”, and vice-versa.)

- Truth Axiom 4:

(4a) $(\forall t_0 \; \exists t \; \exists t' \text{: } t \leq t_0 \leq t', \; T(p,t) \cap T(p,t') \Rightarrow T(q,t) \cap T(q,t')) \Rightarrow ((\forall t_0 \; \exists t \; \exists t' \text{: } t \leq t_0 \leq t', \; T(p,t) \cap T(p,t')) \Rightarrow (\forall s_0 \; \exists s \; \exists s' \text{: } s \leq s_0 \leq s' \; T(q,s) \cap T(q,s')))$

and

$(\forall O \;\; \forall t_0 \;\; \exists t \;\; \exists t' \text{: } t \leq t_0 \leq t', \;\; T(p,t,O) \cap T(p,t',O) \Rightarrow T(q,t,O) \cap T(q,t',O)) \Rightarrow$

$((\forall O \; \forall t_0 \; \exists t \; \exists t' \text{: } t \leq t_0 \leq t', \; T(p,t,O) \cap T(p,t',O)) \Rightarrow (\forall O' \; \forall s_0 \; \exists s \; \exists s' \text{: } s \leq s_0 \leq s' \; T(q,s,O') \cap T(q,s',O')))$

(paraphrase: If at any time t (and according to any observer), “p is true before and after t” implies “q is true before and after t (but at the same times as p)”, then “p is true before and after t, at any time t (and according to any observer)” implies “q is true before and after s, at any time s (and according to any observer) (the times before and after s could be different from those relating to p)".) Note that the converse of this Truth Axiom does **not** hold.

(4b) $(\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', (T(p,t) \cap T(p,t')) \cap (T(q,t) \cap T(q,t'))) \Rightarrow ((\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(p,t) \cap T(p,t')) \cap (\forall s_0\ \exists s\ \exists s': s \le s_0 \le s'\ T(q,s) \cap T(q,s')))$

and

$(\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', (T(p,t,O) \cap T(p,t',O)) \cap (T(q,t,O) \cap T(q,t',O))) \Rightarrow ((\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(p,t,O) \cap T(p,t',O)) \cap (\forall O'\ \forall s_0\ \exists s\ \exists s': s \le s_0 \le s'\ T(q,s,O') \cap T(q,s',O')))$

(paraphrase: If at any time t (and according to any observer), both "p is true before and after t" and "q is true before and after t (but at the same times as p)", then both "p is true before and after t, at any time t (and according to any observer)" and "q is true before and after s, at any time s (and according to any observer) (the times before and after s could be different from those relating to p)".) Note that the converse of this Truth Axiom does **not** hold.

(4c) $((\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(p,t) \cap T(p,t')) \cup (\forall s_0\ \exists s\ \exists s': s \le s_0 \le s'\ T(q,s) \cap T(q,s'))) \Rightarrow (\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', (T(p,t) \cap T(p,t')) \cup (T(q,t) \cap T(q,t')))$

and

$((\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(p,t,O) \cap T(p,t',O)) \cup (\forall O'\ \forall s_0\ \exists s\ \exists s': s \le s_0 \le s'\ T(q,s,O') \cap T(q,s',O'))) \Rightarrow (\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', (T(p,t,O) \cap T(p,t',O)) \cup (T(q,t,O) \cap T(q,t',O)))$

(paraphrase: If either "p is true before and after t, at any time t (and according to any observer)" or "q is true before and after s, at any time s (and according to any observer)" is true, then at any time t (and according to any observer), either "p is true before and after t" or "q is true before and after t" is true.) Note that the converse of this Truth Axiom does **not** hold.

(4d) $(\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', \neg(T(p,t) \cap T(p,t'))) = (\forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(\neg p,t) \cup T(\neg p,t'))$

and

$(\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', \neg(T(p,t,O) \cap T(p,t',O))) = (\forall O\ \forall t_0\ \exists t\ \exists t': t \le t_0 \le t', T(\neg p,t,O) \cup T(\neg p,t',O))$

(paraphrase: If at any time t (and according to any observer), it is not true that we have both "p is true before t" and "p is true after t", then at any time t (and according to any observer), either "p is not true before t" or "p is not true after t" is true. The converse also holds.) Note that this axiom is only added for completeness, as it is simply a deduction of De Morgan's theorem in propositional calculus and the above Truth Axiom 2d.

6.2 Temporal Properties

Based on the above Truth Axioms, the following properties linking our temporal formalization and propositional logic could be proven syntactically.

In the following, we use the symbol '$\Rightarrow$' to denote the implication relation in propositional calculus and also, to simplify the notations we will suppose that c is defined over **P** only, as similar properties could be written when c is defined over **P**x**T** or **P**x**T**x**O.**

For any temporal concept type c in T_C and for any propositions p, q and r in **P**, we have the following properties:

(1) $c(p \Rightarrow q) \models (c(p) \Rightarrow c(q))$

(paraphrase: if "p implies q" is true under a temporal concept type c, then "p is true under c" implies "q is true under c".) For example, if the proposition: "p implies q" has always been true (i.e., the proposition is a Permanent Past truth), then the proposition: "p has always been true" implies the proposition: "q has always been true".

(2) $(p \Rightarrow q) \models (c(p) \Rightarrow c(q))$

(paraphrase: if "p implies q" is true, then for any temporal concept type c, "p is true under c" implies "q is true under c".)

(3) $c(\neg p) \models \neg c(p)$

(paraphrase: For a temporal concept type c, if "non-p is true under c", then it is not true that "p is true under c".) Note that the converse of this property does **not** hold.

(4) $c(p \cap q) \models (c(p) \cap c(q))$

(paraphrase: If the proposition "p and q are true" is true under c (i.e., p and q are jointly true under c), then both propositions: "p is true under c" and "q is true under c" are true.) For example, if both p and q will always be jointly true (i.e., "p and q" is a Permanent Future truth), then "p will always be true" and "q will always be true" are both true.

(5) $(c(p) \cup c(q)) \models c(p \cup q)$

(paraphrase: if either proposition "p is true under c" and "q is true under c" is true, then the proposition "either p or q is true" is true under c.) For example, if either "p will always be true" or "q will always be true" is true, then "p or q is true" will always be true. Note that the converse of this property is not true, e.g., if "p or q is true" is a Discrete Permanent truth, then it is not necessarily true that either "p is a Discrete Permanent truth" or "q is a Discrete Permanent truth " is true, since, at any time t, "p or q" is true before and after t (e.g., p is true before t and q is true after t), but it is not necessarily true that "p is true both before and after t" or "q is true both before and after t".

(6) $\neg(c(p) \cap c(q)) = (\neg c(p) \cup \neg c(q))$

This is an extension of De Morgan's Theorem No. 1 in propositional calculus.

(paraphrase: If it is not true that both "p true under c" and "q true under c" can be jointly true, then it must be true that either "p not true under c" or "q not true under c" is true, and vice-versa.) For example, if we cannot have both "p is a Discrete Permanent truth" and "q is a Discrete Permanent truth", then either "p is not a Discrete Permanent truth" or "q is not a Discrete Permanent truth" is true, and vice-versa.

(7) $(c(\neg p) \cup c(\neg q)) \models \neg(c(p) \cap c(q))$

This is an extension of Temporal Property 6 above.

(8) $\neg(c(p) \cup c(q)) = (\neg c(p) \cap \neg c(q))$

This is an extension of De Morgan's Theorem No. 2 in propositional calculus.

(paraphrase: If it is not true that either "p true under c" or "q true under c" is true, then it is true that "p not true under c" and "q not true under c" are both true.) The converse also holds. For example, if we cannot have either p or q as a Future truth, then we can have both "p not a Future truth" and "q not a Future truth" (i.e., both p and q are not Future truths), and vice-versa.

(9) $(c(\neg p) \cap c(\neg q)) \models \neg(c(p) \cup c(q))$

This is an extension of Temporal Property 8 above.

(10) Temporal Modus Ponens: $c((p \Rightarrow q) \cap p) \models c(q)$ (paraphrase: If both "p implies q" and p are true under c, then q is true under c.) Note that it could be proven that a similar Temporal Modus Ponens formula does **not** hold: $(c(p \Rightarrow q) \cap c(p)) \models c(q)$

(11) Temporal Modus Tollens:

$$c((p \Rightarrow q) \cap \neg q) \models c(\neg p)$$

(paraphrase: If both "p implies q" and "not q" are true under c, then "not p" is true under c.) Note that it could be proven that the similar following Temporal Modus Tollens formulae do **not** hold:

- $c((p \Rightarrow q) \cap \neg q)) \models \neg c(p)$
- $(c(p \Rightarrow q) \cap \neg c(q)) \models c(\neg p)$
- $(c(p \Rightarrow q) \cap \neg c(q)) \models \neg c(p)$

(12) Temporal Transposition:

$$c(p \Rightarrow q) \models (\neg c(q) \Rightarrow \neg c(p))$$

(paraphrase: If "p implies q" is true under c, then it is true that "q not true under c" implies "q not true under c".) Note that Transposition is similar to Modus Tollens, but they are not the same as Modus Tollens emphasizes the non-true value of q in the conclusion while Transposition emphasizes the semantic entailment relation in the conclusion proposition.

(13) Temporal Distribution:

$$(c(p) \cup c(q \cap r)) \models (c(p \cup q) \cap (c(p \cup r))$$

(paraphrase: If "p true under c" or "q and r jointly true under c", then both "p or q true under c" and "p or r true under c" are true.) Note that it could be proven that the following similar formulae do **not** hold:

- $(c(p) \cap c(q \cup r)) \models (c(p \cap q) \cup (c(p \cap r))$
- $(c(p \cup q) \cap \neg c(p)) \models c(q)$
- $(c(p \Rightarrow q) \cap c(q \Rightarrow r)) \models c(p \Rightarrow r)$

We apologize for not being able to include the proofs for the above properties in this paper due to space restriction.

7 Conclusion

This paper proposes a novel formalization of temporal notions in which all objective and subjective temporal concept types are identified, based on McTaggart's A- and B-series and Priorean tense logic, with the help of propositional calculus and first-order logic. Our approach enables categorization of tempo-modal propositions under a time ontology, structured according to a formalism that we previously introduced. In our time ontology, we identify through syntactic proof 32 n-ary subsumption relations among the 33 temporal concept types, forming a hierarchy that could be graphically represented as a tree structure. Some axioms and properties linking our temporal logic with propositional calculus are also identified, contributing to future research in combining time and event ontologies. Possible world semantics and multi-agent systems are other directions that could be explored in the future in conjunction with our concept of subjectivity in time and event. Our ultimate aim is to use our temporal logic to assist formal reasoning involving time, including the development of the Semantic Web, e.g., by describing the temporal content of web pages and by building automated natural language translation engines.

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