Computational Geometric and Combinatorial Approaches to Digital Halftoning

Tetsuo Asano

School of Information Science,
JAIST (Japan Advanced Institute of Science and Technology)
1-1 Asahidai, Nomi, Ishikawa, 923-1292 Japan

Digital halftoning is a technique to convert a continuous-tone image into a binary image consisting of black and white dots. It is an important technique for printing machines and printers to output an image with few intensity levels or colors which looks similar to an input image. In this talk I will explain how computational geometry and combinatorial optimization can contribute to digital halftoning or what geometric and combinatorial problems are related to digital halftoning (Aronov et al. 2004, Asano et al. 2004).

Conventional halftoning algorithms are classified into two categories depending on resolution of printing devices. In a low-resolution printer such as an ink-jet printer individual dots are rather clearly separated. On the other hand dots are too small in a high-resolution printer such as off-set printer to make fine control over their positions. Therefore, dots should form clusters whose sizes are determined by their corresponding intensity levels. Such a halftoning algorithm is called a cluster-dot halftoning.

This algorithm consists in partitioning the output image plane into repetitive polygons called screen elements, usually of the same shape such as rectangles or parallelograms. Each screen element is then filled in by dots according to the corresponding intensity levels. Dots in a screen element is clustered around some center point to form a rounded figure. Denoting by k the area or the number of pixels of a screen element, only $k+1$ different intensity levels instead of 2^k levels are reproduced since the gray level in a screen element is determined only by the number of dots in the region. So, large screen element is required to have effective tone scale. On the contrary the size of a screen element should be small for effective resolution. This suggests a serious tradeoff between effective resolution and effective tone scale. So, it is required to resolve it by introducing adaptive mechanism to determine cluster sizes.

In most of the conventional cluster-dot halftoning algorithms the output image plane is partitioned into screen elements in a fixed manner independent of given input images. A key idea of our algorithm is to partition the output plane into screen elements of various sizes to reflect spatial frequency distribution of an input image. This adaptive method is a solution to balance effective resolution and effective tone scale in the following sense. The two indices are both important, but one is more important than the other depending on spatial frequency distribution of an input image. That is, resolution is more important in a high-frequency part to have a sharp contour, so that the sizes of screen elements should be kept small. On the other hand, tone scale is more meaningful in a low-frequency part with intensity levels changing smoothly, and so larger sizes of screen elements are preferred. All these requirements suggest the following geometric optimization problem. Given a continuous-tone image A and a scaling factor to define the size of an output image, we first compute spatial frequency distribution by applying Laplacian or Sobel differential operator. Then, each grid in the output image plane is associated with a disc of radius reflecting the Laplacian value at the corresponding point. Now, we have a number of discs of various radii. Then, the problem is to choose a set of discs to cover the output plane in an optimal way. The optimality criterion should reflect how large area is covered by exactly one disc from the set, which implies minimization of the area of unoccupied region and intersection among chosen discs to make the resulting screen elements rounded figures.

Optimization of a dither mask used in a so-called Ordered Dither algorithm for halftoning is also an interesting topic. The problem is how to arrange n^2 integers from 0 to n^2-1 as uniformly as possible over an $n \times n$ matrix. Again we introduce a discrepancy-based measure to evaluate the uniformity. The measure is based on the observation that if those integers are uniformly distributed over a matrix then the average of elements in a rigid submatrix (or region) of a fixed size must be the same wherever we take such a submatrix. So, we define the discrepancy of a matrix to be the largest difference of the average in such a region with the average in the whole matrix.

Different schema to achieve low discrepancy for a family of square regions are described. More concretely, we prove that the discrepancy for a family of 2×2 regions can be 0 if and only if the matrix size is even. For families of larger regions there is a scheme to achieve 0-discrepancy for regions of size $k \times k$ in a matrix of size $n \times n$ if the integer k divides n. On the other hand, the discrepancy cannot be 0 if the matrix size n and region size k are relatively prime.

References

Aronov, B., Asano, T., Kikuchi, Y., Nandy, S. C., Sasahara, S. & Uno, T. (2004) 'A Generalization of Magic Squares with Applications to Digital Halftoning' *in* Proc. of 15th International Symposium on Algorithms and Computation, ISAAC 2004, pp. 89-100. To appear in Theory of Computing System.

Asano, T., Katoh, N., Tamaki, H. & Tokuyama, T. (2004) 'The structure and number of global roundings of a graph', Theoretical Computer Science, 325(3):425-437.