Development of Tools for the Study of Chaotic Behavior in Power Electronics

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Abstract - This paper describes the development of software tools for the analysis of chaos in power electronics systems. This project was motivated by potential contributions of chaos theory in the design, analysis and control of power electronics circuits. Nonlinear analysis software and computer programs were used for the detection and analysis of chaotic components. Simulations of power electronics devices were performed using commercially available circuit analysis packages. The voltage and current time series obtained from the circuit analysis were studied using the nonlinear analysis tools developed for this project as well as existing nonlinear analysis programs. The results of the nonlinear analysis can be used to design and implement better mitigating and control techniques for the circuits under study. Chaotic dynamics provide an alternate representation that can lead to better designs and new ways to explain nonlinear behavior in power electronics circuits. There is also a great pedagogical value in complementing the traditional representation of power electronic circuits with alternate models.

I. INTRODUCTION

Power electronics devices have revolutionized the way electric energy is processed and used. These devices provide new ways to control and utilize energy more efficiently [1]. Nowadays, half the electric energy in the U.S. is processed by some kind of power electronics device. However, the nonlinear characteristics of most power electronics circuits may produce power quality problems, e.g., harmonics [2]. Mitigating strategies need to be developed to deal with such undesirable effects. The use of power electronics models in computer simulations enables the researcher to experiment with a wide variety of operating conditions and identify the critical aspects of the device operation that create power quality concerns. Later, the researcher can build a scaled version of the system under study based on the software simulations.

Many traditional methods to model power electronics circuits include the linearization of components [1]. Although, using idealized characteristics simplifies the analysis, leaving out the nonlinearity inherent in power electronics may affect the results of the simulation. Perhaps more important, unexpected difficulties may appear in the implementation of the circuit or mitigating strategy. Chaotic dynamics provide an alternate way to model nonlinearities in power electronics devices. Chaotic systems have time responses that appear to be noise, but the dynamical representation of such processes is deterministic [3]. Previous research work has emphasized the chaotic characteristics in the Buck converter [4] - [6]. Nevertheless, there are chaotic components in other power electronics circuits [7]. Chaos theory can make a significant contribution in control schemes, noise reduction and other power electronics applications [7]. Chaotic dynamics provides an alternate modeling approach that can lead to better designs and new ways to explain some of the nonlinear phenomena in power electronics circuits.

The objective of this paper was to compile available nonlinear analysis and simulation tools for use in power electronics. The motivation for this work was the potential contribution of chaos theory in the analysis and control of power electronics circuits. Algorithms for the calculation of the largest Lyapunov exponent [8], [9] were used to study of chaotic behavior in simple systems such as the Lorenz system [3]. Tools for the study of more complex systems are currently being developed. The simulations of power electronics devices were performed using Spice.

II. CHAOS IN POWER ELECTRONICS

A. Theoretical Background

Voltages and currents in an electronic circuit can behave in one of the following ways: a) settle to a constant value; b) increase exponentially until limited by the power supply; c) vary with a defined period; d) behave chaotically [10]. Chaos is only found in nonlinear systems (of order three or greater), however a common philosophy in circuit analysis is to linearize before analysis. This approach does not account for experimentally observed nonlinear phenomena that can be of paramount importance in power electronics. For example, for certain sets of parameters of dc-dc converters the steady state trajectory does not settle to a desired value but appears to approach an aperiodic waveform. This kind of erratic behavior could be explained using nonlinear models based on chaotic dynamics [10].

Chaotic signals show irregular, non-periodic but bounded behavior. Sensitivity to initial conditions is another characteristic of chaos; even small changes in initial conditions may cause the dynamics of a chaotic system to change significantly. Chaotic time series appear to be random even though

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the system dynamics are known to be deterministic. The present state of a chaotic system has some relationship with previous and future states. Some authors prefer to use the term *deterministic chaos* to emphasize this point. These characteristics make chaotic systems difficult to predict in the long term. If the equations that govern the dynamics of a system are known, the presence of chaos can be mathematically proven, even in the presence of discontinuities [11]. Chaotic dynamics can also be identified from system measurements for some systems [3], [11].

These seemingly contradictory characteristics of chaos can be understood using stability concepts. A system may show unstable properties within a bounded region, while showing stable behavior globally (to an observer outside the bounded region) [3], [12]. Trajectories in a chaotic system are contained within a region of state space, called a strange attractor for its peculiar shape. Variations in initial conditions will cause nearby trajectories to diverge, following one of the orbits contained in the attractor, but the trajectories will remain *inside* a bounded region (the attractor). In other words, a chaotic system is an aperiodic limit cycle [3], [13]. In a chaotic system, it is difficult to predict actual values far ahead. However, the attractor determines global features useful in the analysis of the system. Fig. 1 shows the xy phase portrait of a well-known chaotic system, the Lorenz system [14].

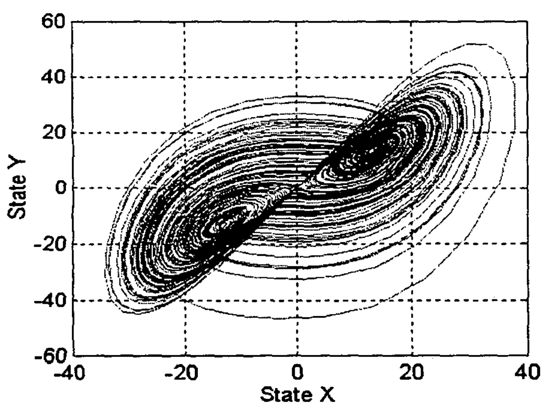

Fig. 1. Attractor for the Lorenz system (xy-state plane)

The presence of chaos can be viewed from two perspectives. Chaotic signals as the cause of undesirable behavior in the system, e.g., non-desired "noise" and unreliability of system operation. The goal on these studies is to detect chaos and find ways of reducing its impact on the system. On the other hand, the chaotic properties of a process can be used in a positive way as a mechanism of identification, a tool for analysis or even to mitigate other problems [15]. For example, if chaotic behavior is part of a mode of operation, it will limit the ability to predict future states of a system. Forecasting windows can be created to identify how far in the future can predictions be made reliably [16].

B. Conditions for Chaos

Chaos in power electronics is sometimes treated as noise, other times it is regarded as a difficult subject with no apparent effect on circuit operation. However, power converters are rich in chaotic dynamics. When the regions of operation are pushed towards stability limits, it is worthwhile to explore if the so-called unstable modes are in fact chaotic. Some of the first references of chaos in power electronics date back to the early 1980s [17]. Even though many references present theoretical studies, chaos has also been reported in practical operating ranges of power electronic devices [7]. Some drive systems as well as several dc-dc converter configurations exhibit chaotic behavior [18] - [22].

The interaction of nonlinear components with certain range of operating parameters can cause qualitative changes or bifurcations in a power converter that lead to multiple steady states. Which steady state emerges will depend on the initial conditions [10]. If bifurcations continue to occur, the converter could start exhibiting chaotic behavior. Common sources of nonlinearities in power electronics include: Non-linear snubber capacitors, the capacitance of diodes, BJTs, and mosfets; non-linear inductance in transformers, ferroresonant controllers and saturable snubber inductors.

Load changes, noise and external disturbances create the need for the control of the converter duty cycle [7]. Most of the literature of chaos in power converters deals with instabilities caused by the operation of switches controlled by pulse width modulation (PWM). Switch-mode systems tend to develop instability and chaos because switches cause the topological structure of the converter to vary with time [10]. In other words, the operation of switches and other nonlinear components makes the converter piecewise-linear in time, introducing discontinuities in the circuit operation [11].

Chaos may appear in current-mode or voltage feedback controller topologies [7], [23]. The use of a switching frequency much higher than the system characteristic frequencies is another cause for chaos in power electronics [24]. Some designs might use a simple circuit to deal with protective modes; however operation of a converter in a protective mode can also lead to chaos [20]. The interaction of a saturated transistor and a diode-resistor combination creates a lack of synchronization between a circuit and its trigger mechanism that may cause chaos [13].

III. SIMULATION TOOLS

Tools commonly used for power electronics simulations can be classified into three groups: Power electronics simulation tools; general transient simulation tools or EMPT type of programs; general harmonics simulation tools or frequency domain simulation tools [25]. For this work, the first classification was considered.

Students developed simulation tools as part of an undergraduate research project guided by a faculty advisor. Commercially-available packages were used to simulate dc-dc converters. As a first step, a simple program that simulates laboratory components was used to study currents and voltages in the circuit components. Because of the ease of its use, this software was used to introduce students to power electronics fundamentals. Spice was used for the actual implementation of power electronic topologies. It was also used to obtain current and voltage waveforms for analysis of chaotic properties.

The goal was to develop modules that could be used to study chaos in various operating modes of the converters. These simulations provided the necessary data for nonlinear analysis. These modules are the first modules of an environment for the study of chaos in power electronics. At this early stage, emphasis was given on topologies that the authors considered present fundamental concepts in power electronics, the Buck and boost converters. Furthermore, there is ample literature coverage on these two topologies.

The time step for the simulations should be carefully selected. It should be small enough to avoid convergence problems and to ensure that chaotic dynamics are not affected. Guidelines to avoid truncation and rounding errors in the simulation of chaos in Spice are presented in [13].

A. BUCK CONVERTER

This topology is also known as a step down converter because its principal function is to transform a non–regulated voltage (input) to a lower level regulated voltage (output). The inductor current is positive, meaning that it is operating in a continuous mode. Applications of the buck topology include resonant converters (reduce switching losses), in ac-ac converters (regulate bus voltages, isolation), and in electronic ballasts (used for power factor correction).

The simulations in this section were based on information cited in [7]-[10], [13] and [22]. Fig. 2 shows the configuration of the Buck converter implemented in Spice. The input-output relationship of this converter is given by

$$V_{out} = V_{in} * D. \qquad (1)$$

Controlling the switch's duty cycle, D, the value of V_{out} can be kept within a desired range. The converter has PWM control of the switching. A reference voltage is compared to the output voltage (capacitor voltage), and then the result is compared to a sawtooth (ramp) waveform so that corrections can be made on the value of D.

A first order Buck converter (no capacitor in the output) was simulated to familiarize students with the topology. Students tested a range of voltages to identify those values that did not cause a quick change in the system while the switch was closed (multiple pulsing). Students also experimented with values of circuit parameters. The second order buck converter (as shown in Fig. 2) exhibits chaotic dynamics as discussed in Section IV.

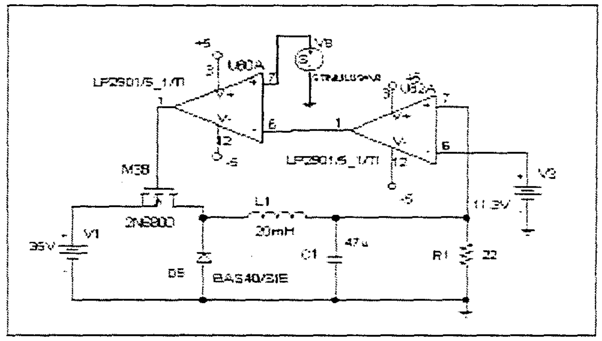

Fig. 2. Buck converter with PWM control

Parameters used were: L = 20 mH, C = 47 μF, R = 22 Ω, amplifier gain = 8.4, V_{ref} = 11.3 v, initial ramp voltage = 3.8 v, upper ramp voltage = 8.2 v, T= 400 μs.

B. BOOST CONVERTER

This topology is also known as a step up converter because the output voltage is greater or equal to the input voltage. There are two modes of operation: Continuous and discontinuous. In continuous mode, the inductor current is always positive. If the inductor current reaches zero before the end of a period, the mode of operation is discontinuous. Discontinuous mode occurs when the inductance is too small or the switching period is relatively long. The value of inductance should be greater than L_{min} in (2) to avoid the discontinuous mode.

$$L_{min} = [D*(1-D)^2*R]/[2f] \qquad (2)$$

where D is the duty cycle, R is the load, and f is the switching frequency.

The PWM control of the switching in the Boost is similar to the structure discussed for the Buck converter. The frequency of the sawtooth waveform was 3 kHz, and the reference sinusoid frequency used was 60Hz. The input-output relationship is:

$$V_{out} = V_{in}/(1-D). \qquad (3)$$

The simulations in this section were based on [15], [20] and [21]. Fig. 3 shows the Boost topology as well as the values used for the simulations.

The duty cycle of the circuit studied was 0.2874, the value of R = 125 Ω. Using these values in (2) results in an L_{min} = 3 mH. The inductor value used was 2 mH, therefore simulations of the Boost converter were made in discontinuous mode. For the intervals where the inductor current is zero, the diode and switch do not allow conduction to the rest of the circuit while. The capacitor discharges through the load. Analysis of the chaotic properties of the boost converter will be presented in the next section.

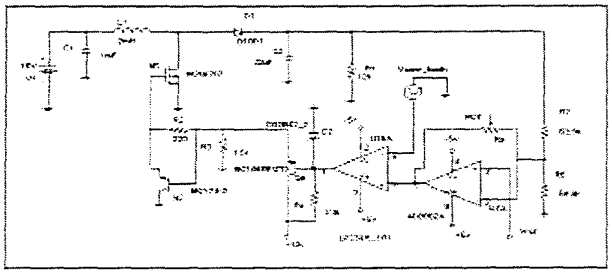

Fig. 3. Boost converter with PWM control

IV. ANALYSIS TOOLS

Within Spice, there are various analytical tools that were used to study the characteristics of the topologies discussed in Section III. Current and voltage time series were also analyzed using Matlab. Time series and frequency domain observations were carried out. The attractor of both converters was studied to identify chaotic properties.

A. Buck Converter

Chaos occurred because the inductor current entered in a non-periodic region as the input voltage of the buck (see Fig. 2) was increased. Input voltage was varied from 30 to 35 volts. If the loop gain is high, the ripple that is fed-back interferes with the PWM process. The voltage is kept close to reference, but the ripple is chaotic [10]. Nevertheless, since chaos is bounded behavior, the circuit can be made to work reliably [13]. Thus, chaotic operation of the converter does not necessarily mean unreliable operation. The capacitor voltage followed the changes in inductor current.

Fig. 4 shows voltage and current in the state space. It shows non-periodic, but bounded behavior, and the dynamics are known to be deterministic. Thus, we have qualitative evidence of chaos in the buck converter. Results obtained from the simulations agree with previous studies in the area.

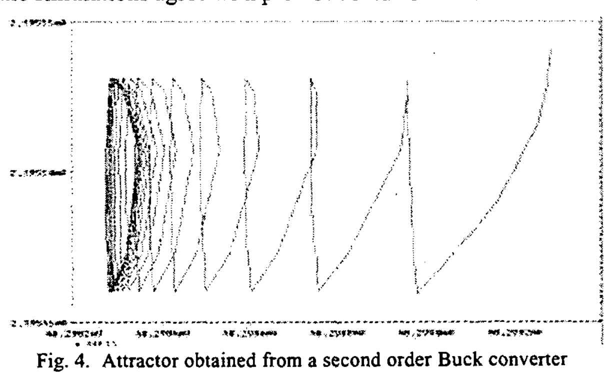

Fig. 4. Attractor obtained from a second order Buck converter

B. Boost Converter

In this simulation, the closed loop gain was varied between 0.09 and 0.15. The close loop gain is given by

$$k_g = R_f/(V_M R_l) \tag{4}$$

where R_f is the variable resistor in Fig. 3, R_l is the resistor at the top of the voltage divider in Fig. 3 and V_M is the height of the ramp voltage applied to the second comparator. The converter exhibited chaos for values over 0.14. Fig. 5 shows the capacitor voltage waveform for k_g=0.14. There is bounded, non-periodic behavior emerging from a deterministic system without any random stimulus. Thus, there is qualitative evidence of chaotic behavior.

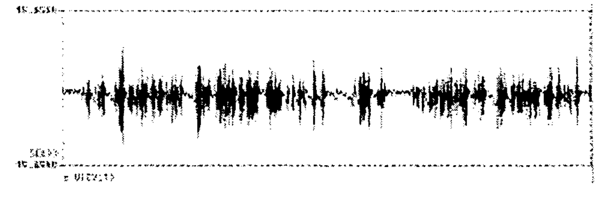

Fig. 5. Time series for capacitor voltage in a boost converter

Table 1 presents the results of the analysis carried out using time series and attractors from the topologies simulated.

TABLE 1
SUMMARY OF RESULTS FOR THE BUCK AND BOOST CONVERTERS

Converter Type	BUCK	BOOST
Operating mode	Continuous	Discontinuous
Affected measures	Inductor current/ Capacitor voltage	Inductor current/ Capacitor voltage
Cause for chaotic behavior	Input voltage range from 30 to 35 volts	Close loop gain, k_g greater than 0.14

C. Lyapunov Exponents

The Lyapunov exponents are a measure of the rate of divergence (or convergence) of state trajectories whose initial conditions are infinitesimally close together. A positive Lyapunov exponent indicates net average divergence from initial conditions. This in turn implies sensitive dependence on initial conditions, and therefore the presence of chaos.

In general, there are k Lyapunov exponents associated with a dynamic process in a k-dimensional phase space. For a discrete dynamical system, the k^{th} exponent is defined as

$$\lambda_k = \ln[\lim_{n \to \infty} (s_{n,k})^{1/n}] \tag{5}$$

whenever the limit exists, $s_{n,k}$ can be regarded as the length of the k^{th} semi-axis of the n^{th} iterate of an infinitesimally small ellipsoid of initial conditions [3]. The Lyapunov exponents are a useful diagnostic tool, since the magnitude of the largest positive Lyapunov exponent determines the time scale over which the system dynamics become unpredictable.

If the dynamics of a system are known, one could apply (5) to obtain all k exponents. If there is no mathematical description of the system, then the largest positive Lyapunov exponent, λ_1, can be estimated using a method based on embedding dimensions [8]-[9]. Chaotic systems that do not contain discontinuities can be easily handled analyzed using either method. However, switching in power electronic devices create discontinuities that complicate the calculation of Lyapunov exponents [11].

Software was developed in Matlab to model and obtain the Lyapunov exponent of simple chaotic systems. The exponents could be regarded as "theoretical" values since they were obtained directly from a model. A program was also written in Fortran to determine the largest Lyapunov exponent from experimental data [8]. The program was validated with the simple models, and its results closely matched the "theoretical" values. However, due to the discontinuities of the simulated data from the buck and boost converters, the algorithm did not converge. Other algorithms are being evaluated to address this problem [11].

Chaos in power electronics should receive attention because it can create undesirable nonlinear effects in some devices. If the regions of chaotic operation are identified, one can avoid those modes of operation. However, chaos might be an inherent part of the operation of a circuit that cannot be filtered out or disregarded. Since chaos is bounded behavior, the chaotic operation of a dc-dc converter does not necessarily mean unreliable operation. The practical merit of studying chaos is a better understanding of the nonlinear dynamics of dc-dc converters, which will lead to more reliable designs, and new possibilities of operating regimes that can help optimize *design [10].*

V. OUTCOMES

Although the study of non-linear systems was not an easy task for the students involved in this project, the experience motivated them to study harder and perform close to graduate level. They engaged in literature search, technical writing, and presentation of results to their peers and industry people. It was also an opportunity to work closer with Faculty. For the advisor it was a chance to integrate his research interests into teaching (undergraduate research is an elective course at the University of Puerto Rico).

Students used the simulation and analysis tools to identify mitigating strategies for chaos. The first recommendation is to reduce switching frequency to diminish the impact of parameter changes due to chaos. A designer should also strive to keep circuits as simple as possible because complex switching schemes result in more nonlinearities and potential sources of chaotic behavior. Finally, it is very important to test components before implementation in order to identify possible imperfections that may cause nonlinear problems.

Besides the research value of these tools, there is great pedagogical value in complementing the traditional representation of power electronic circuits with alternate models. This work provides an excellent teaching tool to demonstrate not only power electronics principles, but also the use of nonlinear dynamics in a practical setting. This paper also demonstrates the value of interdisciplinary work, since we would be applying a seemingly complex mathematical subject to the solution of an engineering problem. These tools can be used to familiarize electrical engineering students with nonlinear analysis, and also mathematics and physics students with the engineering applications of nonlinear dynamics.

VI. FUTURE WORK

The ultimate objective of this research is to develop a software environment in which to study, not only chaos, but nonlinear phenomena in general. This project is the foundation for the subsequent development of power electronics experiments (hardware) that would support undergraduate and graduate courses in power electronics, power quality and possibly mathematics or physics. The tools discussed in this paper constitute a first step in the compilation of resources to be used in an applied nonlinear systems laboratory at the University of Puerto Rico-Mayagüez (UPRM).

Work presently under way includes the implementation of other topologies such as the buck-boost and the Cuk converters. In regard to the calculation of the Lyapunov exponents, a variational approach discussed in [11] is being considered to deal with the discontinuities of power electronics measurements.

Further research using these tools could result in: Potential reduction of EMC (by chaotic spread spectrum) [15][26]; better understanding of the stability of circuits [15]; mitigation and control of chaos [27], [28]; controlled perturbations of a chaotic system to obtain a wider range of operating modes (flexibility of chaotic systems) [28]. However, there are certain guidelines to follow when purposely using the chaotic properties of a converter. In [29], the authors have identified the regions of operation to ensure reliable chaotic operation (robust chaos) and a boost topology was tested satisfactorily.

VII. CONCLUSIONS

The study of chaos may yield a better understanding of the nonlinear dynamics of dc-dc converters, which may result in reliable designs, and new possibilities of operating regions that can help optimize design. The analysis tools developed in this project will be used to demonstrate power electronics principles in the classroom. Simulations were developed using Spice. The calculation of Lyapunov exponents in power electronics was also considered.

The development and testing of software tools to study chaos is a significant contribution to research efforts in power electronics at the University of Puerto Rico-Mayaguez (UPRM). This approach provides an alternate representation that can lead to new ways to describe some of the nonlinear phenomena in power electronics circuits.

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