Observational Heterarchy as Phenomenal Computing

Yukio-Pegio Gunji & Moto Kamiura

Department of Earth & Planetary Sciences, Faculty of Science, Kobe University Nada Kobe 657-8501 JAPAN

yukio@kobe-u.ac.jp

Abstract

We propose the notion of phenomenal computing as a dynamical pair of a computing system and the environments of executing computation. It is expressed as a formal model of observational heterarchy inheriting robustness against structural crisis. Observational heterarchy consists of two different categories connected by pre-adjoint functors where inter-categories operations are defined as pre-functors. Owing to the attribute of prefunctor, the model reveals robust behaviors against perpetual structural changes.

1. Introduction

Whenever one argues regarding autonomy, emergent property, and/or consciousness, one adheres to examine them in terms of binary opposition such as the computable vs non-computable. In that perspective every computing is defined as a process with its algorithm, and computing is separated from its environment to execute computing. The environment of computing cannot be separated from computing in real computing process. Imagine a working computer in a small close box. Since the inner temperature in the box increases due to the work of computer, the computer can output errors. The working computer influences its environment and the environment can entail to erroneous computing. When one takes such an evolution of computing into consideration, one has to grasp a whole phenomenon consisting of computing and its environment. We call the computing with the environment to execute computing phenomenal computing (Gunji, 2003; Gunji et al., 2003a,b).

Phenomenal computing consists of two levels that are different with respect to logical status and inherits the mixing them up. Previously we propose a model of phenomenal computing by a dynamical infomorphism (Gunji et al., 2004a,b). We generalize the idea of phenomenal computing as observational heterarchy inheriting emergent property and robustness, and propose a categorical model by defining pre-functor that is an engine leading the mixture of intra- and inter-levels operations.

Copyright © 2004, Australian Computer Society, Inc. This paper appeared at the *Computing and Philosophy Conference*, Canberra. Conferences in Research and Practice in Information Technology, Vol. 37. J. Weckert and Y. Al-Saggaf, Eds. Reproduction for academic, not-for profit purposes permitted provided this text is included.

2. Heterarchy

Heterarchy is a dynamical hierarchical system of which an action at one level simultaneously reveals reactions at other levels. The significance of heterarchy is manifesting with respect to difference of stability and robustness (Jen, 2003). Although heterarchy and/or hierarchical systems are thought to be real matters that can be found in particular systems (e.g., human society), we here show that heterarchy is derived from internal measurement (Matsno, 1989; Gunji, 1993, 1995, Gunji et al., 1997a,b;2002). That is why any systems are regarded as heterarchy. Internal measurement is defined by description with reservation. If one observes a system within a system, one has to accept the constraint of observation. The internal observer cannot overlook a whole system. As a result, his observation and/or description are always accompanied with reservation, and it is mixed with the outside of description. In an abstract sense such a mixture entails to the collapse of consistent description. The description with reservation is expressed as an inconsistent dynamical system.

First we start to describe a heterarchy of human being. A man is not only a member of his family but one of the company to which he employed. His action, therefore, affects both family and company, simultaneously. Imagine going to the company on holiday. Although such an action is good for his boss, it is bad for his family. If you listen to this topic, you might think that it satisfies the condition of heterarchy, simultaneous interaction among levels. You have to, however, notice that such a simultaneous interaction results just from a hidden specific operation such that bad (or good, respectively) for a family is mapped to good (bad, respectively) for the company. One cannot recognize "simultaneousness" in interaction, till one comprehends both independent two levels (family and company) and simultaneous interaction. Because of independency of two levels, one has to take all possible operations between two levels into consideration. Moreover, one has to focus on the process of choice of one operation. Now, define a set of value for the family and the company as $S = \{0(bad), 1(good)\}$. We call all possible operations from the family to the company Interpretaion-0, -1, -2, and -3. Operations are defined by the follows.

Interpretaion-0: $0 \rightarrow 0$; $1 \rightarrow 0$, Interpretaion-1: $0 \rightarrow 1$; $1 \rightarrow 1$, Interpretaion-2: $0 \rightarrow 0$; $1 \rightarrow 1$, Interpretaion-3: $0 \rightarrow 1$; $1 \rightarrow 0$.

An observer has to describe a man's action—going to the company on a holiday—as a simultaneous process of choosing one interpretation. What is a simultaneous process? It makes sense if a chosen interpretation has a

value of *S*. The situation is, actually, described by the following.

On a holiday a man intends to go to the company and wears his shoes at an entrance, where his son and wife who expected to go to the zoo are angry. The husband is hesitating to go to the company, and is thinking that going to the company is bad for a family but is good for his boss. The thinking (i.e., choosing Interpretation-3) is proceeding in a finite time at the entrance. Therefore, such a process itself can have the value in *S*, in a family. A wife begins to feel that her husband is suffering from going to the company, and to thinks that her own attitude gives him too much feeling of guilt. She thinks that she should make a smile to her husband who goes to the company. As a result, she told him good-by with a smile against her anger.

The husband's hesitation proceeds at an entrance. It means that choosing an interpretation makes sense even in a family, and that triggers emergence of new value, "smile against anger" within *S*. As a result, the value in a family changes from $\{0, 1\}$ to $\{0, 1, 2(\text{smile against anger})\}$. What choosing an interpretation makes sense in a particular level (family) changes the structure of the level.

We here generalize such a process by the following. Heterarchy is defined by simultaneous interaction among some levels. It is replaced by the simultaneous choice between intra-level dynamics and inter-level dynamics. In the example of going to the company, the intra-level dynamics is just a choice of a value of S (i.e., a value of a particular level) and the inter-level dynamics is a choice of an interpretation from Interpretation-0~3. The simultaneous choice is defined by two properties; (1) a map-property, and (2) simultaneous making value. The map-property is defined by; for all elements of S, there exists an interpretation. For example, for 0 in a family, a husband chooses Interpretation 3, and for 1 he chooses Interpretation 1. It makes a map. By contrary, if for 0 he chooses both Interpretation-3 and -4, a map is destined to be one-to-many and the map property is collapsed. The second property is defined by the following. Each possible chosen interpretation has to have a value in a level (e.g., family). The map-property looks natural however it needs all possible correspondence between an element of S and all interpretations. Even if one observes only one correspondence between 0 and interpretation 1, an observer has to decide the correspondence for 1 because of the map-property. The simultaneous making value is defined so as to expand such a stance. Imagine that a map is defined by;

 $0 \rightarrow$ Interpretation-3, $1 \rightarrow$ Interpretation-1.

Simultaneous choice requires that each interpretation has a value of S in this choice. For this choice, one can recognize that Interpretation-1 has a value 1 and Interpretation-3 has a value 0, as soon as an each interpretation is chosen. However, the property of simultaneous making value requires making value for all interpretations. Although Interpretation-0 and -2 are not chosen, they also have to have values of S. Assume that Interpretation-0 has a value 0. If so, the map-property is collapsed because a value-0 is mapped both to Interpretation-0 and -3. As a result, the map-property and simultaneous making value constitutes a trade-off relationship.

If each level is defined by a set, *S*, a set of the inter-level operations are defined by Hom(*S*, *S*) that is a set of functions from *S* to *S*. The-map property of simultaneous choice is defined by; $f : S \rightarrow \text{Hom}(S, S)$ is a map. Moreover, the property of making value is defined by; *f* covers all elements of Hom(*S*, *S*). As a result, simultaneous choice requires that a map *f* is surjective (i.e., for all elements *y* of co-domain of *f*, there is an element *x* in domain of *f* such that y=f(x)). Such a requirement is fallen to fail, in principle. Denote the number of elements of *S* as *N*. The number of elements of Hom(*S*, *S*).

However simultaneous choice is collapsed, heterarchy proceeds as a real system. In this situation, one has to focus on the notion of heterarchy as a real system against the collapse of observer's framework. Remember that an example of going to the company on a holiday. A proceeding motion against the collapse happens in this example. The appearance of emergent state "smile against angry" can be explained by a happening proceeding against the collapse. The situation of which choice of interpretation also makes sense in a family is expressed as an assumption of a surjective map from S to Hom(S, S)(i.e., the map requires simultaneous choice) (Fig. 1A). If one attempts to make a system satisfy simultaneous choice in spite of the collapsed assumption, one has to find new source that is mapped to possible elements of Hom(S, S) out of S. In Fig. 1A, a map called choice from S to Hom(S, S) is drawn as a thin arrow, and emergent arrows required by simultaneous choice is drawn as thick arrows. In order to avoid one-to-many mapping, a new source of an arrow is constructed out of {good, bad}. It is nothing but new state of a family, such as "smile against angry". The collapsed assumption called simultaneous choice makes re-organization of the system possible. It is the engine of heterarchy.



Fig. 1. Heterarchy consisting of family and company. Fig. 2. Evolution of observational heterarchy.

Finally, we define a heterarchy as the following.

Definition 1 (Heterarchy)

If a system consists of two different subsystems, intrasubsystem operations and inter-subsystems operations, and if the mixture between intra- and inter-operations is permitted, the system is called heterarchy.

Economic system between two countries satisfies the definition of heterarchy. Given two countries, Japan and Australia, each country is expressed as a map by which goods are evaluated in a term of currency (Yen or Australian dollar). These maps represent intra-subsystem operations. There is an exchange between two maps, and that represents inter-subsystem operation. In real economic systems, the exchange itself is regarded as a good (i.e., the exchange of currencies needs cost), and that leads to the mixing between inter-levels operation and intra-level operation. That is why the system is heterarchy.

3. Observational Heterarchy

The next question arises whether heterarchy is a systemic property attribute to an agent who can make a decision, or not. In the example of going to the company on a holiday, an agent is a man who can make a decision referring to a company and/or society. Making decision inherits plural levels in its own right. Our answer to the question is no. Even for a population of proteins in a cell, an observer has to describe it as a heterarchy. In other words, the internal measurement is introduced by a description of an object as heterarchy. We call it observational heterarchy.

First we define two perspectives, intent-perspective and extent-perspective. Given a system (phenomenon, concept), intent-perspective is defined by attribute of a system, and extent-one is defined by a collection of objects to which the system can be applied. As for any systems in a set theory, intent perspective is equivalent to extent one. For example, intent of even number (concept) is expressed as 2n, and extent of it is expressed as $0, 2, 4, \dots$ As a result a pair of intent and extent, and generalized intent and extent are defined by the following.

Definition 2 (Generalized Intent and Extent)

Given a concept, Intent is defined as a collection of attributes of the concept, and Extent is defined as a collection of objects to which the concept is applied. Conversely, given two collections of attributes and objects, if each object has all attributes and each attribute contributes to all objects, a pair of collections is called a pair of Intent and Extent, and we say that intent and extent constitutes a concept. The operations by which an attribute in intent is applied to an object in extent are called inter-level (or inter-) operations. A triplet, <intent, extent, inter-level operation> constitutes a concept.

Given two sets of maps, such as Hom(X, X') and Hom(Y, Y') (i.e., intra-level operation is defined as a map, $f:X \rightarrow X'$ or $g:Y \rightarrow Y'$) and the inter-level operations such that F: $\text{Hom}(X, X') \rightarrow \text{Hom}(Y, Y')$ and G: $\text{Hom}(Y, Y') \rightarrow \text{Hom}(X, X')$ where F(X), F(X'), and F(f) (G(Y), G(Y'), and G(g), respectively), if a map, $\text{Hom}(F(X), Y') \rightarrow \text{Hom}(Y, G(Y'))$ is bijective, we call Hom(X, X') and Hom(Y, Y') generalized intent and generalized extent, respectively. Generalized intent (extent, respectively) is also called intent (extent, respectively) is also called intent (extent, respectively) perspective. A triplet < Hom(X, X'), Hom(Y, Y'), $\langle F, G \rangle$ constitutes an adjunction or generalized concept.

For a general system, however, two perspectives are inconsistent with each other. Imagine that you observe a behavior of a population of proteins in a cell. On one hand, you try to describe it as a generalized attribute or function. You describe a state of population as a concentration, and a dynamics in a term of the concentration as a differential equation. That is intentperspective of a behavior of the population. On the other hand, you describe detailed dynamical structure of each protein that is folded and has a three dimensional structure. The three dimensional structure of a protein is dependent on local hydrogen bond and the cluster of water molecules. Therefore, it varies dependent on local distributions of various molecules. In this sense, you have to evoke difference and/or different objects in the form of a protein. You describe dynamics of a population in focusing on an individual reactions of proteins carrying various three dimensional structures. That is extentperspective of a behavior of the population. If these two perspectives are equivalent, the one can be replaced with the other. It results in a model of the population in the form of a differential equation in a term of the

concentration of the protein. In such a case, it is easy to see that intent and extent perspective constitutes a particular concept for a particular protein.

However, equivalence between intent- and extentperspectives is just approximation as for a behavior of the population of the protein. The extent that is inconsistent with intent-perspective is latent. Adaptive mutation (Shapiro, 2002) illustrates the appearance of latent property. Splitting enzyme for sugar is controlled by operon on DNA. If it switches on, the enzyme is elaborated, off, not. In the experiment of adaptive mutation, E coli bacteria are cultured in culture media with sugar. The DNA of bacteria is converted not to elaborate the splitting enzyme corresponding to the sugar. The bacteria are wasted because of absence of the enzyme, and that gives rise to malfunction of DNA-protein system. Mutation rate becomes high, and mutation hits the broken gene corresponding the splitting enzyme for the sugar. As a result, the bacteria can acquire the ability to use the sugar as energy source.

We can describe intent-perspective of a population of the enzyme controlled by DNA as a differential equation. To simplify the situation, we express the concentration of the enzyme by two states $\{0(\text{increasing}), 1(\text{decreasing})\}$. Extent-perspective is expressed as a set of switches revealing the three dimensional structure of proteins and tactile process. It is also expressed as a set of two states $\{0(on), 1(off)\}$. In the condition with usable energy source, the system described by intent-perspective is functioned well, and that implies intent-perspective is equivalent to extent-perspective. It is, however, just approximation. The simultaneous choice between intraand inter-level (translation between two levels) is latent in such a normal condition. The simultaneous choice appears under the condition with absence of energy source (i.e., with unusable sugar). Inter-level operations cannot be ignored, and then simultaneous choice explicitly appears as well as the case of going to the company on a holiday. As a result, a map form $\{0, 1\}$ to Hom($\{0,1\}$, $\{0,1\}$) must be surjective, and gives rise to emergence of new state out of $\{0(on), 1(off)\}$. It means the outside of adequate function of switches either on or off, and that is instabilizing DNA and i.e., mutation with high rate. It gives rise to adaptive mutation, and it stops till DNA-protein system functions well (Fig. 2).

Observational heterarchy is defined by the following.

Definition 3 (Observational Heterarchy)

A pair of generalized intent and extent (or intent and extent perspectives) is called heterarchy if and only if the inter-operation between intent and extent inherits the mixture of the approximated inter-operation and the intralevel operation, where a triplet <generalized intent, generalized extent, the approximated inter-operations> constitutes a generalized concept.

From definition 3, a population of a particular protein is an observational heterarchy. If one neglects latent evoluvability of the protein, he obtains a triplet <generalized intent, generalized extent, inter-operations>, where generalized intent is expressed as a collection of functions of proteins, and generalized extent is expressed as a collection of structures of individuals of proteins, and inter-level operations are expressed as consistent relationship between intent and extent. Since the evoluvability resulting from indefinite environment latently exits, such a generalized concept is just an approximation. If one pays attention to evoluvability, inconsistency between generalized intent and extent can appear. That is why it is an observational heterarchy.

Observational heterarchy is summarized as the following. (1) Heterarchy consists of two levels and inter-level operations. (2) Simultaneous interaction among levels is defined as *simultaneous choice* that is expressed as surjective map from a set of one level to a set of interlevel operations. (3) Simultaneous choice implies the collapse of the logical framework, and then heterarchy is regarded as a system inheriting logical collapse. (4) Owing to the logical collapse, heterarchy gives rise to reorganization of the structure. (5) Heterarchy is not a real entity but it results from the interaction between an object and an observer. Two levels are essentially intent- and extent-perspectives. In the next section, we propose the abstract system of observational heterarchy.

4. Active Coupling as Observational Heterarchy

The toy model of heterarchy is proposed, by illustrating a cell-cell interaction. The essential property of heterarchy is simultaneous choice that is expressed as the mixture of intra-level and inter-level, bringing emergent property. Robustness and emergent property are both sides of the same coin. Motion against the collapse of logical framework can be demonstrated as a robust heterarchy. For this purpose, we define two-levels, intent- and extent-perspectives as two categories in terms of category theory. A category consists of objects (each object has its own identity arrow) and arrows (composition of arrows satisfy associative law). Inter-level operations are defined by adjunctive functors leading to equivalence between two levels. Under this framework, we define pre-functor and pre-adjunction. It leads to dynamics of heterarchy.

Assume that two cells are coupled by diffusion-like material transportation. Each cell follows chaotic dynamics called a logistic map, and only a small amount of materials are flowed from the other cell. Therefore, a cell consists of major part originated from its own material, and minor part transported from the other cell. Assume that major part can be described as intentperspective and minor part has to be described as extentperspective. Two perspectives are connected by two functors from transportation map, $f: A \rightarrow B$. Intent-and extent-perspectives are defined by a comma category C/Aand C/B, respectively. If two functors are defined by composition functor, $f: C/A \to C/B$, and pull-back functor, $f: C/B \to C/A$, equivalence between two categories can be obtained, such that $C/B(f(X|A), Y|B) \cong C/A(X|A, f(Y|B))$ (1)

Under the framework, we define pre-functor. A functor, F is defined as an operator between two categories so as to preserve composition of arrows and identity, such that for

an object, X, F(X) and for an arrow $g:X \rightarrow Y$, F(g). Preservation of composition is expressed by F(gh)=F(g)F(h). We define pre-functor by $F(g) = fgf^*$, where ff^* can be approximated to an identity. Imagine that objects are sets and arrows are functions. In this case, if f is bijection, $f^* = f^{-1}$, and for a general map, ff^* cannot be an identity. As for a pre-functor, $F(g)F(h) = fgf^*fhf^*$, and then as far as ff^* is an identity and is canceled, $fgf^*fhf^* = fghf^* = F(gh)$. Therefore, pre-functor weakens the preservation of composition.

Second, we define pre-adjunction. Given a pair of adjunctive functors leading that $C(F(X), Y) \cong D(X, T(Y))$, we define two pre-functors such that $F(g)=fgf^*$ and $T(h)=tht^*$. Assume that objects are sets and arrows are maps. Given $f:X \rightarrow F(X)$ and $t:Y \rightarrow G(Y)$, pre-adjunction is defined by; for all x in X and y in Y, $hf(x)=y \Leftrightarrow g(x) = t(y)$. We also introduce the commutative diagram induced from $C(F(X), Y) \cong D(X, T(Y))$, such that by using universal arrow, λ and μ , $g'=tht^*\mu$ and $h'= fgf^*\lambda$. Because $h \neq h'$ and $g \neq g'$, it means time development from g, h to g' and h'. As a result, dynamics of heterarchy is defined.

Because of pre-functor, we introduce the mixture of interlevel operation (functor) and intra-level operation (arrow). An application of a pre-functor to an arrow is defined by composition of arrows. It avoids the direct collapse of the logical framework because of giving up the preservation of composition. Recall the discussion of heterarchy. The mixture between inter- and intra-level operations gives rise to the re-organization of heterarchy against the logical collapse, and that means emergent property. By contrast, the model with pre-functor and pre-adjunction can give rise to robust behavior of heterarchy against the logical collapse.



Fig. 3.Relationship between f (represented by F) and f^* . Fig. 4. Time development of intent- and extent-dynamics

Recall the adjunction between composition and pull-back functor revealing isomorphism, (1). By applying preadjunction to the case, we define pre-functor, < f > such that for an arrow g in C/B, $< f > (g) = f^*gf$. Finally, time development of heterarchy is expressed by;

where $f:A \rightarrow B$ with A=B=[0, 1] is a non-linear nonmonotonous map that can be approximated to $f(x) \approx cx$. The cell-cell interaction is defined by

$$x_i^{t+1} = (1-c) h^t(x_i^t) + g^t(x_j^{t+1})$$
(3)

where *i* and *j* are 0 or 1 and $i \neq j$. Initial condition is given such that $h^0(x) = \alpha x(1-x)$ with a parameter in the chaotic region. If f(x) = cx, we obtain that $x_i^{t+1} = (1-c) h^t(x_i^t) + ch^t(x_i^{t+1})$ representing conventional cell-cell coupling.

Fig. 3 shows a pair of f and f^* , constituting a pre-functor. If f is a non-linear map, f^* must be discontinuous map. Therefore, a pair of intent-perspective, $h^t(x^t)$ and extentperspective, $g^t(x^t)$ is transformed into a pair of discontinuous map (Fig. 4). Such a discontinuity leads to robust coupling. A fixed point of a map, h(x)=x is instable if |h'(x)|<1. If $h(x) = \alpha x(1-x)$ is a chaotic map, differential coefficient of iterated map at a fixed point must be larger than 1. Therefore, many fixed points are instable. By contrast, the model of heterarchy expressed as a pair of discontinuous maps give up instable fixed point. As a result, it gives rise to robust coupling (entrainment) in spite of chaotic dynamics and perpetual change of dynamics (Gunji et al., 2004c).



 $\alpha = 4.0; \quad c = 0.65, 0.675, 0.70, 0.725$

Fig. 4 shows the difference between active coupling and passive coupling with respect to riddled basin structures of synchronization, where passive coupling is conventional cell coupling such as $x_i^{t+1} = (1-c) h(x_i^t) +$ ch (x_j^{t+1}) . A co-ordinate in Cartesian product [0,1]×[0,1](each rectangle) represents an initial condition such as (x_0^0, x_1^0) , and if two states are synchronized after *T* steps (i.e., $x_0^T = x_1^T$), the co-ordinate is painted white, and otherwise, painted black. In the regions of these coupling strengths ($c = 0.65 \sim 0.725 < 0.75$) two states are finally synchronized, however, the transitional behaviors are different from each other. In active coupling, synchronization is achieved more rapidly than the case in passive coupling. Especially, even in the chaotic regions of the coupling strength (c < 0.25 or 0.75 < c) active can reveal synchronization, however coupling conventional passive coupling reveals only chaotic behaviors.

The scheme of active coupling is defined by a triplet, $\langle C/B(Y/B, Y/B'), C/A(X/A, X/A'), < f, < f \rangle >>>$. Since $\langle \rangle$ can be approximated by a pull-back functor f_i the approximated triplet such as $\langle C/B(Y/B, Y/B'), C/A(X/A, X/A'), < f, \rangle >>>$ constitutes adjunction or generalized concept due to the isomorphism (1). By contrast, a prefunctor $\langle \rangle >$ inherits the mixture between intra-operation (arrow) and inter-operation (functor). That is why a triplet $\langle C/B(Y/B, Y/B'), C/A(X/A, X/A'), < f, < f \rangle >>>$ is an observational heterarchy.

5. Conclusion

We propose how to recognize a system consisting of a computer and the environment of executing computation, called phenomenal computing. It consists of two levels that are different from each other with respect to logical status. Because of the difference, mixing with two levels leads to logical collapse or emergence. Such a property is also illustrated by heterarchy that is a dynamical hierarchical structure. We claim that heterarchy is not a real entity but a phenomenon resulting from the interaction between an object and an observer. It is called observational heterarchy. In the framework of observational heterarchy, it consists of intent- and extentperspectives and the synchronous choice between intralevel dynamics and inter-levels one is inherited in heterarchy. Observational heterarchy inherits the ability of evolution, robustness and/or adaptability resulting from the synchronous choice. We generalize phenomenal computing as observational heterarchy.

We illustrate the observational heterarchy by a cell-cell interaction. In the model, major and minor parts of the population of materials are described by intent- and extent-perspectives, respectively. Each perspective is described as a category constituting adjunction, and interlevels operations are re-expressed as pre-functor that is defined so as to abandon the conservation of composition of arrows. Pre-functor reveals the mixture of intra-level dynamics and inter-levels dynamics, in weakening a functor. As a result, it leads to perpetual change of structure. In spite of structural instability, two cells are entrained with each other. Because a pre-functor reveals both continuous and discontinuous map induced from a transportation map, it modifies a intra-level dynamics to a discontinuous map. As a result, instable fixed point is perpetually abandoned, and that reveals robust behaviors. It illustrates essential mechanism of phenomenal computing or observational heterarchy showing robustness against logical collapse.

References

- Jen, E. (2003). Stable or robust? What's the difference ? *Complexity* **8**, 12-18.
- Gunji, Y-P. (1993). Form of life : Unprogrammability Constitutes the outside of a system and its autonomy. *Appl. Math. & Comp.* **57**, 19-76.
- Gunji, Y-P.(1995). Global logic resulting from disequilibration process. *Biosystems* **35**, 33-62.
- Gunji, Y-P. & Toyoda, S. (1997a) Dynamically changing interface as a model of measurement in complex systems. *Physica*D101:27-54.
- Gunji, Y-P., Ito, K. & Kusunoki, Y. (1997b) Formal model of internal measurement: alternate changing between recursive definition and domain equation. *Physica*D110:289-312.
- Gunji, Y.-P., Kusunoki, Y. & Aono, M. (2002) Interface of global and local semantics in a self-navigating system based on the concept lattice. *Chaos, Solitons & Fractals* **13**(2), 261-284.
- Gunji, Y.-P., *Generative Life: Theory of Life II.* Tetsugaku-shobou (Philosophical Press), Tokyo, 2003 (in Japanese).
- Gunji, Y.-P. Miyoshi, H. Takahashi, T. & Kamiura, M. Dynamical duality of type- and token-computation as an abstract brain. *Chaos, Solitons & Fractals* (2004a, to appear)
- Gunji, Y.-P., Takahashi, T. & Aono, M., Dynamical infomorphism: form of endo-perspective. *Chaos, Solitons & Fractals* (2004b, to appear).
- Gunji, Y.-P. & Kamiura, M. Observational heterarchy enhancing active coupling. *Physica* D (submitted, 2004c).
- Matsuno, K. *Protobiology: Physical Basis of Biology.* CRC Press, Boca Raton, 1989.
- Shapiro, J.A.,(2002) A 21st century view of evolution, *J. Biol. Phys.* **28**,745-778.